CS4618 Artificial Intelligence I | **Thomas Jansen**
Term 2012/2013, 1st Period | **University College Cork**
30 November, 2012

Sample Test

The time for the test is 50 minutes. Answer all questions. The maximum number of marks is 20 (translating to 10 marks in the final grade).

Name: **Student #:**

Topic I) Task Environments (2 marks)

Give a PEAS description for an adaptive traffic light control at an intersection where two streets intersect. Classify the corresponding task environment using the list of seven dimensions from the lecture.

Topic II) Rational Agents (2 marks)

Compare a table-driven agent to a model-based agent.

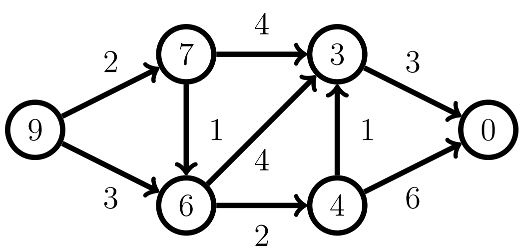
Topic III) Uninformed Search (2 marks)

Remember the 8 puzzle. Consider using BFS or DFS for solving this puzzle and compare the two algorithms.

Topic IV) Informed Search (2 marks)

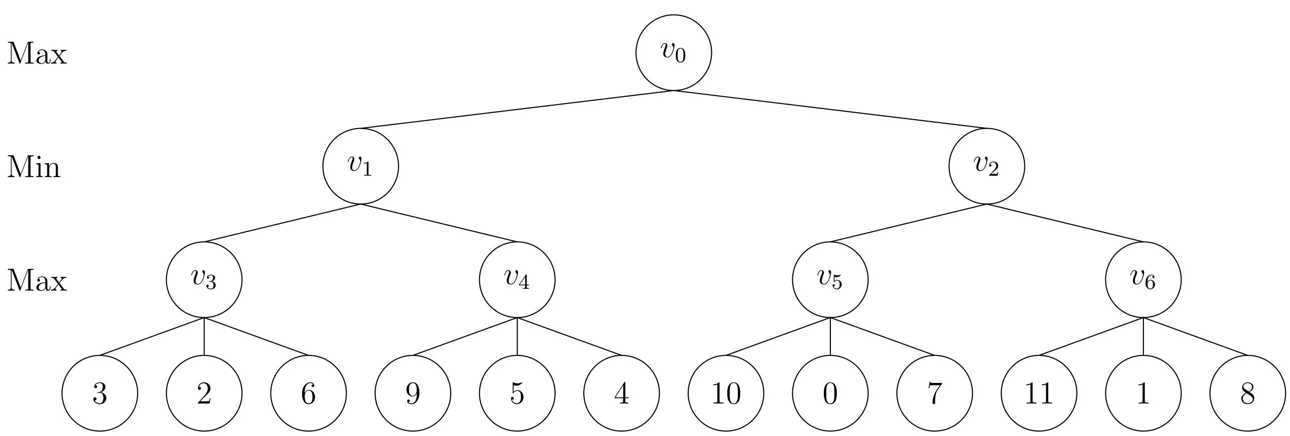
The graph below specifies a search problem. States are nodes, the start state is 9, the only goal state is 0. Edges describe actions and their outcome, the number on an edge gives its cost. We use the number in the nodes as heuristic.

For each node write next to it the minimal cost to reach the goal state from there. Decide if this heuristic is admissible. Decide if this heuristic is consistent.



Topic V) Alpha-Beta Pruning (2 marks)

Evaluate the game tree given below using alpha-beta pruning. Consider the moves from left to right. Write down a list of the states that are not considered. Write next to each state that is evaluated the value that is computed by alpha-beta pruning.



Topic VI) Adversarial Search (2 marks)

How are cut-off tests used to speed up the evaluation of games trees? Name consequences of introducing cut-off tests.

Topic VII) Random Processes (2 marks)

Consider the coupon collector's scenario with n different types of coupons. Assume that you want to collect at least m copies of each type of coupon. Let T be the number of coupons collected until your collection is complete. Prove $\mathrm{E}(T) = \Theta(nm + n \log n)$.

Topic VIII) Black Box Complexity (2 marks)

We call a function $f\colon \{0,1\}^n \to \mathbb{R}$ a function of unitation if its function value for x only depends on the number of 1-bits in x and not x itself. Let S denote the set of all functions of unitation where the function value is strictly increasing with the number of 1-bits. Prove an upper bound on the black-box complexity of S.

Topic IX) Applying Randomised Search Heuristics (2 marks)

Consider attempting to solve a difficult optimisation problem with some evolutionary algorithm. Discuss advantages and disadvantages of using crossover.

Topic X) Analysing Evolutionary Algorithms (2 marks)

Consider the (1+1) EA with mutation probability $p_m = 1/n$ on the function $f\colon \{0,1\}^n \to \mathbb{R}$ below. Prove an upper bound on the expected optimisation time $\mathrm{E}\left(T_{(1+1)\ \mathrm{EA},f}\right)$.

$$f(x) = \begin{cases} n+i & \text{if } x = 1^i 0^{n-i} \text{ with } i \in \{0,1,2,\dots,n\} \\ n - \textsc{OneMax}(x) & \text{otherwise} \end{cases}$$

'Cheat Sheet' (Part 1)

Markov inequality for $X \geq 0$ random variable; $s \in \mathbb{R}^+$: $\mathrm{Prob}\left(X \geq s \cdot \mathrm{E}\left(X\right)\right) \leq 1/s$

Chernoff bounds for $X_1, X_2, \dots, X_n \in \{0,1\}$ independent random variables; $X = \sum\limits_{i=1}^{n} X_i$:

$\forall \delta > 0$: $\mathrm{Prob}\left(X > (1+\delta) \cdot \mathrm{E}\left(X\right)\right) < \left(\frac{e^\delta}{(1+\delta)^{1+\delta}}\right)^{\mathrm{E}(X)}$

$\forall 0 < \delta < 1$: $\mathrm{Prob}\left(X < (1-\delta) \cdot \mathrm{E}\left(X\right)\right) < e^{-\mathrm{E}(X)\delta^2/2}$

Martingale $Y_0, Y_1, Y_2, \dots \in \mathbb{R}$ is a martingale with respect to a random process $X_0, X_1, X_2, \dots$ if the following hold for all $n \in \mathbb{N}_0$.

1. Y_n is a function of $X_0, X_1, \dots, X_n$
2. $\mathrm{E}\left(|Y_n|\right) < \infty$ or $Y_n \geq 0$
3. $\mathrm{E}\left(Y_{n+1} \mid X_0, X_1, \dots, X_n\right) = Y_n$

‘Cheat Sheet’ (continued)

Optional Stopping Theorem Let $Y_0, Y_1, Y_2, \ldots$ be a martingale with respect to $X_0, X_1, X_2, \ldots$. Let T be a stopping time of $X_0, X_1, X_2, \ldots$. If $\exists k \in \mathbb{N}_0 \colon T \leq k$ almost surely or $T < \infty$ and $\exists k \colon |Y_t| \leq k$ for all $t < T$ almost surely then $\mathrm{E}(Y_T) = \mathrm{E}(Y_0)$.

Gambler’s ruin Consider two gamblers, one with initial funds s_A, winning in each round 1 with probability p_A, the other with initial found s_B, winning in each round 1 with probability $p_B = 1 - p_A$. They keep playing until one player runs out of money, i. e., is ruined. Let $p_A \neq p_B$, $q := p_B/p_A$.
$\mathrm{Prob}\,(\text{A is ruined}) = (q^{s_A} - q^{s_A+s_B}) \,/\, (1 - q^{s_A+s_B})$
$\mathrm{E}\,(\text{rounds they play}) = ((1 - \mathrm{Prob}\,(A \text{ ruined}))(s_A + s_B) - s_A) \,/\, (p_A - p_B)$

Collecting coupons Let T denote the number of coupons one collects until one has at least one coupons of in total n different types. The coupons are obtained one by one, each type with equal probability.
$\mathrm{E}\,(T) = n \ln(n) + O(n)$
$\forall \beta > 1 \colon \mathrm{Prob}\,(T > \beta n \ln n) \leq n^{-(\beta-1)}$
$\forall c \in \mathbb{R} \colon \mathrm{Prob}\,(T > n \ln n + cn) = 1 - e^{-e^{-c}}$

number of non-empty classes of function $\mathcal{F} \subseteq R^S$ that are closed under permutation of the search space equals $\left(2^{\binom{|S|+|R|-1}{|S|}} - 1\right) / \left(2^{|R|^{|S|}} - 1\right)$

ANFL theorem Let $S = \{0,1\}^n$, $R = \{0, 1, \ldots, N-1\}$, $f \colon S \to R$, A a randomised BBA. The number of functions $f' \colon S \to R \cup \{N\}$ sucht that A does not find an optimum of f' within $2^{n/3}$ f-evaluations with probability at least $1 - 2^{-n/3}$ is bounded below by $N^{2^{n/3}-1}$.
Of these exponentially many have the additional property that their complexity (measured by evaluation time, circuit size, or Kolmogoroff complexity) is by $O(n)$ larger than that of f.

f-based partitions For $f \colon \{0,1\}^n \to \mathbb{R}$ and $k \in \mathbb{N}$ the sets $L_0, L_1, \ldots, L_k$ are an f-based partition iff the following hold.

1. $\bigcup\limits_{i=0}^{k} L_i = \{0,1\}^n$, $\forall i \neq j \colon L_i \cap L_j = \emptyset$
2. $\forall i \neq j \colon \forall x \in L_i, y \in L_j \colon (i < j) \Rightarrow (f(x) < f(y))$
3. $L_k = \{x \in \{0,1\}^n \mid f(x) = \max\{f(y) \mid y \in \{0,1\}^n\}\}$

Metropolis algorithm

1. $t := 1$; Choose $x_t \in S$ uniformly at random.
2. Select $y \in N(x_t)$ uniformly at random.
3. $t := t + 1$; With prob. $\min\{1, e^{(f(y)-f(x_{t-1}))/T}\}$ set $x_t := y$, else $x_t := x_{t-1}$.
4. If ‘not stop’ continue at line 2.
5. Output x_t.

Steady state GA

1. Choose $x_1, x_2, \ldots, x_\mu \in \{0,1\}^n$ uniformly at random.
2. With probability p_c
   Select $z_1, z_2 \in \{x_1, x_2, \ldots, x_\mu\}$ u. a. r., $z := \text{crossover}(z_1, z_2)$
3. Else select $z \in \{x_1, x_2, \ldots, x_\mu\}$ uniformly at random.
4. $y :=$ standard bit mutation(z)
5. Select new $x_1, x_2, \ldots, x_\mu$ out of best $x_1, x_2, \ldots, x_\mu, y$.
6. If ‘not stop’ continue at line 2.
7. Output x_i with $f(x_i) = \max\{f(x_j) \mid 1 \leq j \leq \mu\}$.