

Assignment 7

Topic ‘Unimodal Functions’

In the following let $k \in \mathbb{N}$ and $n \in \mathbb{N}$ such that n is a multiple of k . We consider bit strings $x_1, x_2, \dots \in \{0, 1\}^n$. We define a sequence S_k^n of such bit strings. We use the usual notation for the concatenation of bit strings: We write b^k for the concatenation of k bs. Thus, e. g., $0^2 1^4 0^3 = 001111000$.

We define S_k^n in the following way. For $n = 0$, we define S_k^n as the empty bit string. For $n > 0$ we assume that the sequence $S_k^{n-k} = x_1, x_2, \dots, x_l$ is already defined. (Note that $n - k \geq 0$ since n is a multiple of k .)

$$S_k^n = 1^k x_1, 1^k x_2, \dots, 1^k x_l, 1^{k-1} 0 x_l, 1^{k-2} 0^2 x_l 1^{k-3} 0^3 x_l, \dots, 10^{k-1} x_l, 0^k x_l, 0^k x_{l-1}, \dots, 0^k x_1$$

Problem 1 Write down the sequence P_3^6 explicitly. Prove that the sequence P_k^n contains exactly $k \cdot 2^{n/k} - k + 1$ bit strings.

Problem 2 For the following function consider the sequence $S_k^n = x_1, x_2, \dots, x_l$.

$$f_{S_k^n}(x) = \begin{cases} n + i & \text{if } x = x_i \text{ for some } x_i \text{ in } S_k^n, \\ \text{ONEMAX}(x) & \text{otherwise.} \end{cases}$$

Prove that $f_{S_k^n}$ is unimodal.

Problem 3 Implementation Task

Remember the implementation of the random local search and the (1+1) EA from Assignments 4 and 6. Implement $f_{S_k^n}$ as objective function. Compare the (1+1) EA and random local search on $f_{S_3^n}$ for $n \in \{9, 18, 27, 36, 45\}$. In each run, let each algorithm run until it finds the global optimum. Average the number of steps taken over several runs. Can you explain your results?