

## Assignment 6

### Topic ‘Collecting Coupons’

**Problem 1** Reconsider the coupon collector’s scenario. We make the following changes: We assume that the number of different coupons  $n$  is even. Initially the coupon collector is given a set of  $n/2$  different coupons in a starter pack.

Clearly, the goal is still to collect all  $n$  different types of coupons. We still assume that each of the  $n$  different types of coupons is obtained with equal probability  $1/n$ . Let  $D$  denote the number of coupons the collector needs to buy before having a complete collection of all  $n$  different types of coupons.

Determine  $E(D)$ .

### Topic ‘No Free Lunch Theorem’

**Problem 2** For a bit string  $a = a_1a_2 \cdots a_n \in \{0, 1\}^n$  let  $\bar{a}$  denote its bit-wise complement of  $a$ . Consider the function  $f_a: \{0, 1\}^n \rightarrow \{0, 1\}$  defined by the following equation:

$$f_a(x) = \begin{cases} 1 & \text{if } x = a \text{ or } x = \bar{a} \\ 0 & \text{otherwise} \end{cases}$$

Consider the class of functions  $\mathcal{F} = \{f_a \mid a \in \{0, 1\}^n\}$ . Is  $\mathcal{F}$  closed under permutation of the search space?

### Problem 3 Implementation Task

Remember the implementation of the Metropolis algorithm from Assignment 4. If you have done this, add the (1+1) EA with mutation probability  $1/n$  to this implementation. If you have not done it, only implement the Metropolis algorithm and the (1+1) EA.

Compare the (1+1) EA and the Metropolis algorithm with  $T = 0.1$  on NEEDLE for  $n \in \{3, 4, 5, \dots, 20\}$ . In each run, let each algorithm run until it finds the global optimum. Average the number of steps taken over 100 runs. Compare the averages for the two algorithms by plotting them in the same diagram (using logarithmic scale for the  $y$ -axis). How do your results relate to the NFL theorem?

NEEDLE:  $\{0, 1\}^n \rightarrow \{0, 1\}$  is defined by

$$\text{NEEDLE}(x) = \begin{cases} 1 & \text{if } x = 111 \cdots 1, \\ 0 & \text{otherwise.} \end{cases}$$