Coláiste na hOllscoile Corcaigh, Éire
University College Cork, Ireland

CS4618 Artificial Intelligence I

Today: Parametrisation: Mutation Probability

Thomas Jansen

December 7th

Plans for Today

Setting the Mutation Probability

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- $p_m = 1/n$ is by far most recommended choice

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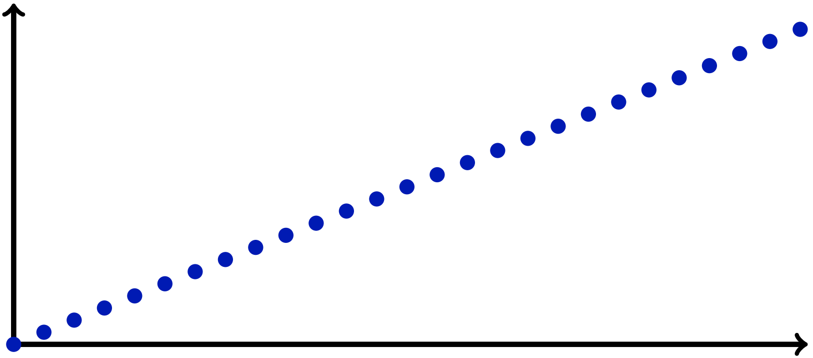
Goal for today develop a more detailed understanding of the mutation probability's role using $\textsc{OneMax}$ has a simple and typical example funct as a starting point

OneMax

$$\text{OneMax}(x) = \sum_{i=1}^{n} x[i]$$



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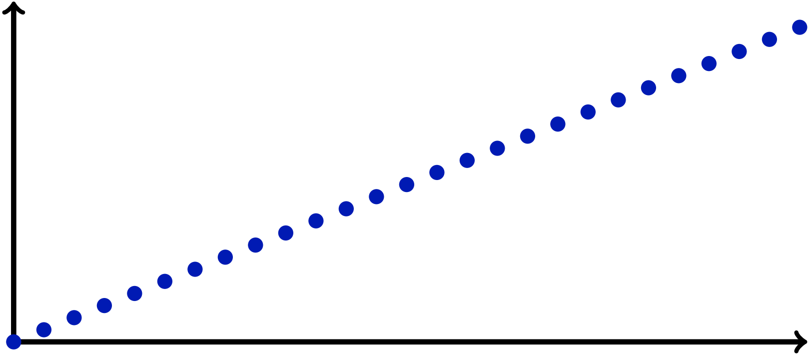

Goal derive simple upper and lower bounds on $\mathrm{E}\left(T_{(1+1)\ \mathrm{EA},\text{OneMax}}\right)$ depending on p_m

A Simple Upper Bound

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$\rightsquigarrow$ only jumps of size $O(\log n)$ feasible

A First Example: $\textsc{Jump}_k$

$$\textsc{Jump}_k(x) = \begin{cases} n - \textsc{OneMax}(x) & \text{if } n-k < \textsc{OneMax}(x) < n \\ k + \textsc{OneMax} & \text{otherwise} \end{cases}$$

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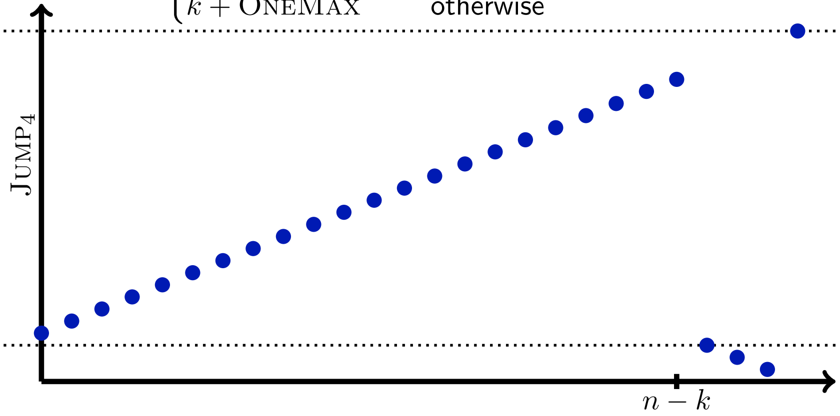

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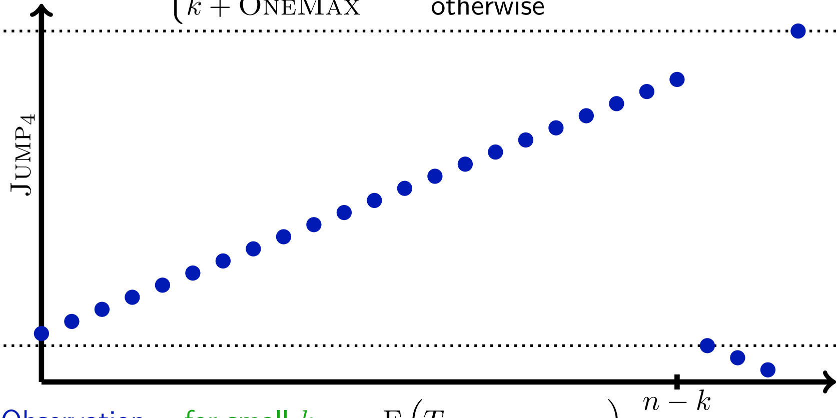

Observation for small k $\quad \mathrm{E}\left(T_{(1+1)\text{ EA},\text{JUMP}_k}\right)$

$$= \mathrm{E}\left(T_{(1+1)\text{ EA},\text{ONEMAX}}\right) + \mathrm{E}\left(\text{time for } k\text{-bit jump}\right)$$

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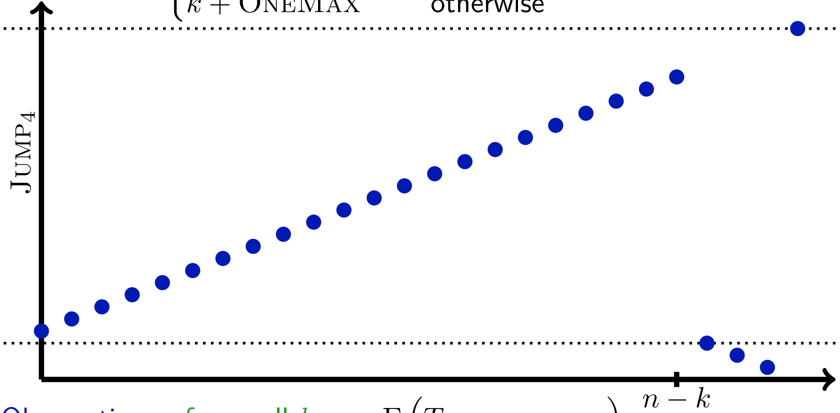

Observation for small k

$$\mathrm{E}\left(T_{\text{(1+1) EA},\textsc{Jump}_k}\right)$$
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$$= O\left(e^{\alpha(n)-\log\alpha(n)} \cdot n\log n\right) + \Theta\left((n/\alpha(n))^k\right)$$

Setting p_m for $\textsc{Jump}_k$

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Observation with standard choice $p_m = 1/n$

$$\mathrm{E}\left(T_{(1+1)\ \text{EA},\text{JUMP}_k}\right) = \Theta\left(n^k\right) \text{ for small } k > 1$$

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Where is the problem?

Observation 'target' very small

Idea larger p_m advantageous for hitting larger targets in some distance

A More Extreme Example

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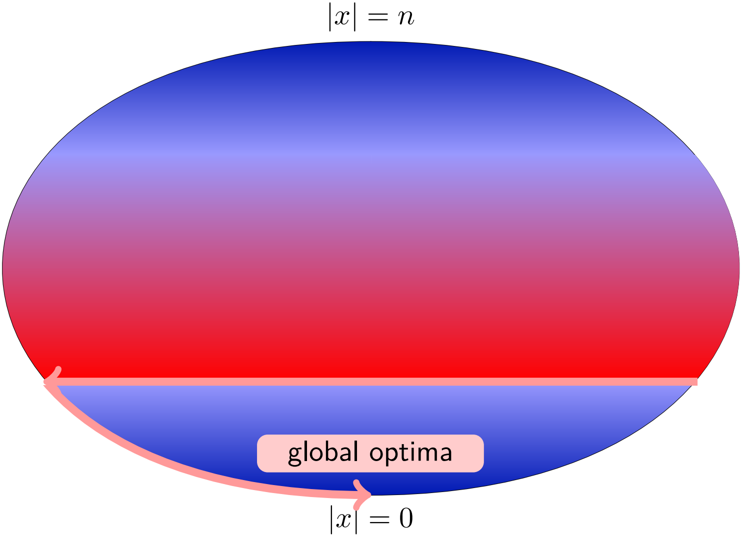
$$f(x) = \begin{cases} n - |x| & \text{if } x \in A \\ (3/4)n + \sum_{i=1}^{n/4} x[i] & \text{if } x \in B \\ 2n - i & \text{if } x \in C \text{ and } x = 1^i 0^{n-i} \\ 2n + 1 & \text{if } x \in D \\ \min\{|x|, n - |x|\} & \text{if } x \in E \end{cases}$$

with

$$A = \{x \in \{0,1\}^n \mid n/4 < |x| < (3/4)n\}$$
$$B = \{x \in \{0,1\}^n \mid |x| = n/4\}$$
$$C = \{x = 1^i 0^{n-i} \mid i \in \{0, 1, \ldots, (n/4) - 1\}\}$$
$$D = \left\{x \in \{0,1\}^n \mid (|x| = \log n) \wedge \left(\sum_{i=1}^{2\log n} x[i] = 0\right)\right\}$$
$$E = \{0,1\}^n \setminus (A \cup B \cup C \cup D)$$

The Example Function f_2



Mutation Probabilities for f_2

Theorem

$$\mathrm{E}\left(T_{\text{(1+1) EA},f_2}\right) = n^{O(1)} \Leftrightarrow p_m = \Theta\left(\frac{\log n}{n}\right)$$

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Proof Ideas

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Proof Ideas

- for $p_m = o(\log(n)/n)$ time for final jump super-polynomial

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- for $p_m = \omega(\log(n)/n)$ time on ridge super-polynomial

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- for $p_m = \omega(\log(n)/n)$ time on ridge super-polynomial
- E (time for final jump with $p_m = c\log(n)/n$)
  $$= \left(\binom{n-2\log n}{\log n}\left(\frac{c\log n}{n}\right)^{\log n}\left(1-\frac{c\log n}{n}\right)^{n-\log n}\right)^{-1}$$

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Remark $\mathrm{E}\left(T_{\text{(1+1) EA},f_2}\right) = O(n^{2.361})$ for $c = 0.361$

But how can we find such a strange good p_m in practice?

A Simple Mutation Schedule

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Idea get rid of the choice of p_m

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by systematically trying all sensible values
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5. $p_m := 2p_m$; If $p_m > 1/2$ then $p_m := 1/n$
6. Until 'decide to stop'
7. Output x_t.

About the Dynamic (1+1) EA

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Observations

- tries $\log n$ different mutation probabilities

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- on many functions (in particular, on simple functions) slower by a factor $\Theta(\log n)$

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- $\exists f \colon \mathrm{E}\left(T_{\text{Dynamic (1+1) EA},f}\right)$ exponential and $\mathrm{E}\left(T_{\text{(1+1) EA } (p_m = 1/n),f}\right)$ polynomial

Summary & Take Home Message

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- fixed mutation probability $p_m = 1/n$ often useful

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