ucc
Coláiste na hOllscoile Corcaigh, Éire
University College Cork, Ireland

CS4618 Artificial Intelligence I

Today: Parametrisation: Offspring Population Size & Mutation Probability

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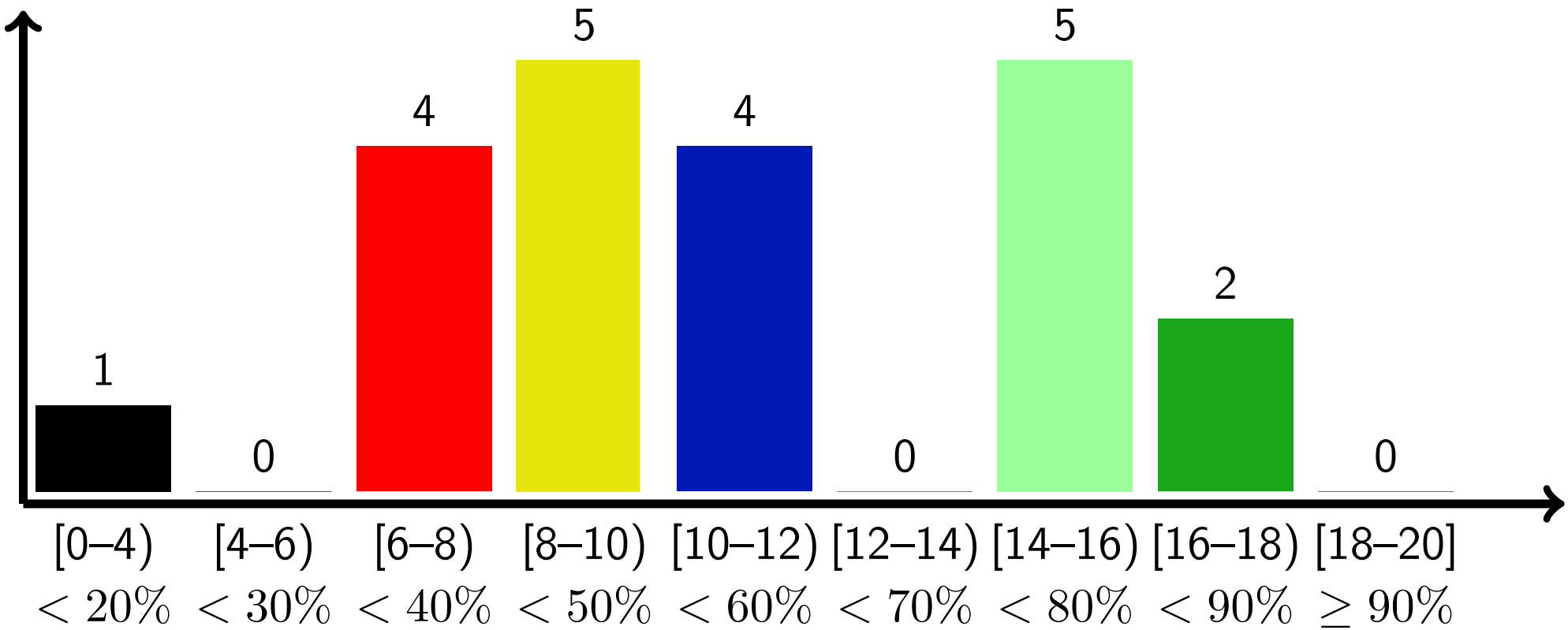
December 5^{th}

Plans for Today

Results for the In-Class Test

more personal details available from me on request

Result Distribution



Statistics

median	11 ≙ 55%		average	11 ≙ 55%
minimum	4 ≙ 20%		maximum	17 ≙ 85%

Topic I) Task Environments

Give a PEAS description for a sorting machine at a recycling facility. The machine is supposed to sort glass bottles with respect to the colour of the glass: clear, brown, green. Classify the corresponding task environment using the list of seven 'dimensions' from the lecture.

- performance measure number of correctly sorted bottles per minute
- environment bottles and parts of the machine
- actuators robot arm or strong air blow or ...
- sensors camera (or possibly only a light sensor)
- properties fully observable, single agent, stochastic, episodic, dynamic, continuous, known

min 1 median 2 average 1.76 max 2

Topic II) Rational Agents

Compare a table-driven agent to a simple-reflex agent.

table-driven agent

- stores list of percepts ($\widehat{=}$ complete history)
- has table mapping lists of percepts to actions

\+ able to solve every task

– table usually huge, possibly infinite $\rightsquigarrow$ not practical

simple-reflex agent

- has table mapping single percepts to actions
- uses no/little memory

\+ simple, efficient

– restricted, not able to solve all problems

min 0 median 2 average 1.62 max 2

Topic III) Uninformed Search

Remember Knuth's number game. Consider using BFS or DFS for solving this puzzle and compare the two algorithms.

BFS

- \+ complete ($\widehat{=}$ guaranteed to find a solution)
- \+ optimal (since all actions have equal cost)
- − may still take a long time, run out of memory, ...

DFS

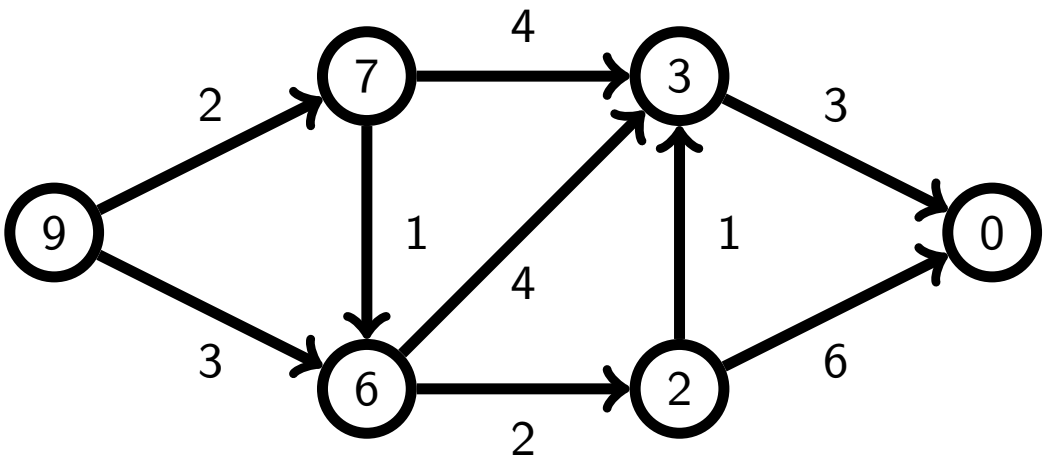
- \+ may be lucky and more efficient than BFS
- − not complete, likely to run indefinitely without success

Consequence prefer BFS over DFS

min 0 median 2 average 1.69 max 2

Topic IV) Informed Search

The graph below specifies a search problem. States are nodes, the start state is 9, the only goal state is 0. Edges describe actions and their outcome, the number on an edge gives its cost. We use the number in the nodes as heuristic. For each node write next to it the minimal cost to reach the goal state from there. Decide if this heuristic is admissible. Decide if this heuristic is consistent.

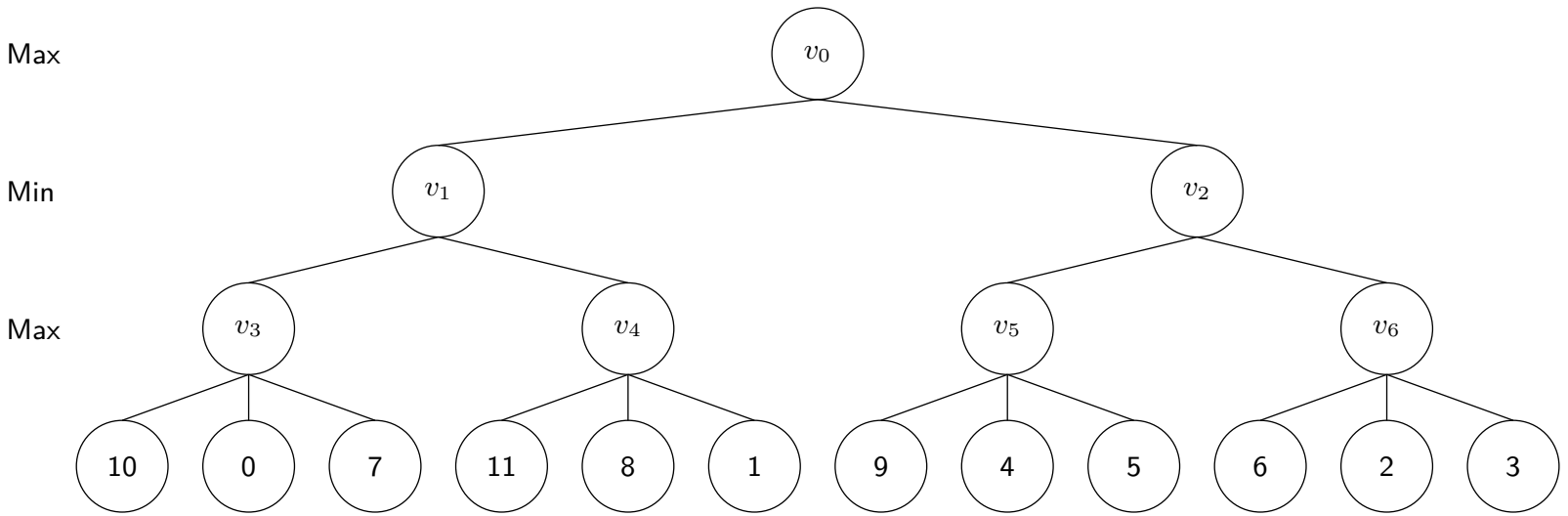


- cost equal to heuristic, except for 2 with cost 4
- admissible since no overestimation
- not consistent since $h(6) > 2 + h(2)$

min 0 median 1 average 1.05 max 2

Topic V) Alpha-Beta Pruning

Evaluate the game tree given below using alpha-beta pruning. Consider the moves from left to right. Write down a list of the states that are not considered. Write next to each state that is evaluated the value that is computed by alpha-beta pruning.



- values v_3: 10, v_4: 11, v_1: 10, v_5: 9, v_2: 9, v_0: 10
- not explored 8, 1, v_6, 6, 2, 3

min 0 median 1 average 1.26 max 2

Topic VI) Adversarial Search

What is a transposition table and why is it useful?

- hash table for states
- helps to avoid re-evaluation of states

min 0 median 1 average 1 max 2

Topic VII) Random Processes

Consider the coupon collector's scenario with n different types of coupons. Assume that you are satisfied with your collection when you have 90% of all different coupons. Let T be the number of coupons collected until your collection contains at least 90% of all different types of coupons. Prove an upper bound on $\mathrm{E}(T)$.

Remember $\mathrm{Prob}(\text{get next new type when having } i) = (n-i)/n$

Thus
$$\mathrm{E}(T) = \sum_{i=0}^{(9/10)n-1} \frac{n}{n-i} = n \sum_{i=(n/10)+1}^{n} \frac{1}{i}$$
$$= n\left(\left(\sum_{i=1}^{n} \frac{1}{i}\right) - \left(\sum_{i=1}^{n/10} \frac{1}{i}\right)\right) = n\left(\ln(n) - \ln(n/10) + O(1)\right)$$
$$= n\left(\ln(n) - \ln(n) + \ln(10) + O(1)\right) = O(n)$$

min 0 median 1 average 0.76 max 2

Topic VIII) Black-Box Complexity

We call a function $f\colon \{0,1\}^n \to \mathbb{R}$ a function of unitation if its function value for x only depends on the number of 1-bits in x and not x itself. Let U denote the set of all functions of unitation. Prove an upper bound on the black-box complexity of U.

Observation There are $\leq n+1$ different function values and we get all by sampling 0^n, 10^{n-1}, 110^{n-2}, $\ldots$, 1^n.

Consequence $\mathcal{B}_U \leq n+1$

min 0 median 0 average 0.1 max 2

Topic IX) Applying Randomised Search Heuristics

Consider attempting to solve a difficult optimisation problem with random local search. Discuss advantages and disadvantages of increasing the neighbourhood from the set of all Hamming neighbours to all points with a Hamming distance of at most 2.

- \+ the increased neighbourhood can help to escape from local optima
- • one can still get suck in some local optima
- – on average good Hamming neighbours found much later
  $\rightsquigarrow$ slower on simple problems/in easy parts of the search space

min 0 median 1 average 1.05 max 2

Topic X) Analysing Evolutionary Algorithms

Consider the (1+1) EA with mutation probability $p_m = 1/n$ on the function $f\colon \{0,1\}^n \to \mathbb{R}$ below. Prove an upper bound on the expected optimisation time $\mathrm{E}\left(T_{(1+1)\text{ EA},f}\right)$.

$$f(x) = \begin{cases} \textsc{OneMax}(x) & \text{if } \textsc{OneMax}(x) \text{ is even or } \textsc{OneMax}(x) = n \\ 0 & \text{otherwise} \end{cases}$$

- use f-based partitions, one level per function value
- for $f(x) = 0$ it suffices to flip any bit
  $\rightsquigarrow s_0 \geq n \cdot (1/n) \cdot (1-1/n)^{n-1} \geq 1/e$
- for $f(x) = v$ it suffices to flip two of the $n - v$ 1-bits
  $\rightsquigarrow$
  $s_v \geq \binom{n-v}{2} \cdot (1/n)^2 \cdot (1-1/n)^{n-2} \geq (n-v)(n-v-1)/(2en^2)$

Thus $\mathrm{E}\left(T_{(1+1)\text{ EA},f}\right) \leq e + \sum_{i=1}^{n/2} \frac{2en^2}{(2i)(2i-1)} < e + 2en^2 \sum_{i=1}^{n/2} \frac{1}{i^2} = O(n^2)$

min 0 median 1 average 0.71 max 2

Remember Results for LeadingOnes

Theorem

$T_{(1+\lambda)\text{ EA},\text{LeadingOnes}} = \Theta\left(n^2 + \lambda \cdot \frac{n}{\log(1+\lambda/n)}\right)$ for any $\lambda = n^{O(1)}$

$G_{(1+\lambda)\text{ EA},\text{LeadingOnes}} = \Theta\left(\frac{n^2}{\lambda} + \frac{n}{\log(1+\lambda/n)}\right)$ for any $\lambda = n^{O(1)}$

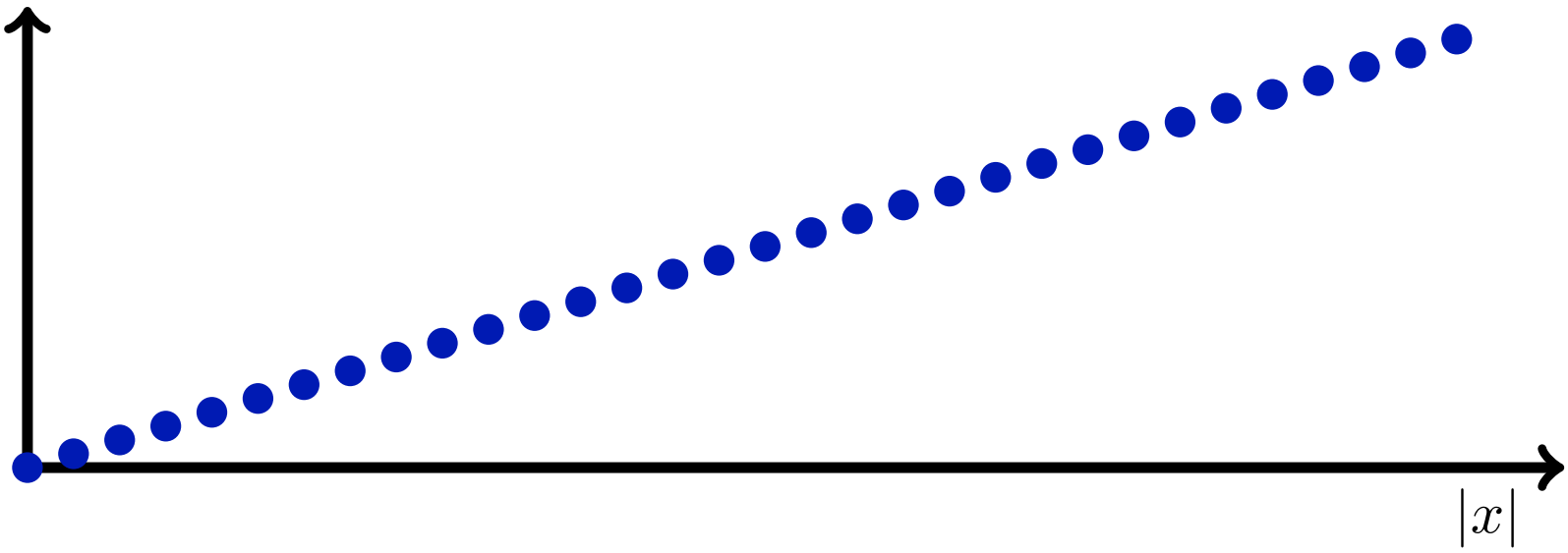
Observations

- $G_{(1+\lambda)\text{ EA},\text{LeadingOnes}} = \Omega(n/\log n)$ for all λ
- $T_{(1+\lambda)\text{ EA},\text{LeadingOnes}} = \Omega(n^2)$ for all λ
- $T_{(1+\lambda)\text{ EA},\text{LeadingOnes}} = O(n^2)$ for $\lambda = O(n)$
- $G_{(1+\lambda)\text{ EA},\text{LeadingOnes}} = \Theta(n)$ $\lambda = \Theta(n)$
- $T_{(1+\lambda)\text{ EA},\text{LeadingOnes}} = \Theta(n^2)$ $\lambda = \Theta(n)$
- with $\lambda = \Theta(n)$ linear speed-up without increased workload

Observation $\text{Prob}\,(\text{single mutation is improving}) = \Theta(1/n)$ always
$\widehat{=}$ reciprocal of $\Theta(n)$

OneMax

Remember $\text{OneMax}(x) = \sum_{i=1}^{n} x[i]$



Prob (single mutation is improving)?

Observation $\text{Prob}\,(\text{single mutation is improving}) = \Theta\left(\frac{n-\text{OneMax}(x)}{n}\right)$

depends on search point, changes during search

(1+λ) EA on OneMax

Theorem

$$\left.\begin{aligned} T_{(1+\lambda)\text{ EA},\textsc{OneMax}} &= O(n \log n) \\ G_{(1+\lambda)\text{ EA},\textsc{OneMax}} &= O\left(\tfrac{n \log n}{\lambda}\right) \end{aligned}\right\} \text{for } \lambda = O\left(\frac{\log(n) \log\log n}{\log\log\log n}\right)$$

$$\left.\begin{aligned} T_{(1+\lambda)\text{ EA},\textsc{OneMax}} &= \omega(n \log n) \\ G_{(1+\lambda)\text{ EA},\textsc{OneMax}} &= \omega\left(\tfrac{n \log n}{\lambda}\right) \end{aligned}\right\} \text{for } \lambda = \omega\left(\frac{\log(n) \log\log n}{\log\log\log n}\right)$$

Observation again linear speed-up without increased workload if λ is sufficiently large but not too large

$\widehat{=}$ with appropriate λ
parallel (1+λ) EA outperforms (1+1) EA
even for very simple fitness function

Setting the Offspring Population Size

Remember

- $(1+\lambda)$ EA
- $T_{(1+\lambda)\text{ EA},f} \mathrel{\widehat{=}}$ number of function evaluations
  $\mathrel{\widehat{=}}$ computational effort
- $G_{(1+\lambda)\text{ EA},f} \mathrel{\widehat{=}}$ number of generations
  $\mathrel{\widehat{=}}$ parallel computation time using $\geq \lambda$ processors
- result on LeadingOnes: $G_{(1+\lambda)\text{ EA},\text{LeadingOnes}}$ decreases without increasing $T_{(1+\lambda)\text{ EA},\text{LeadingOnes}}$ for $\lambda = O(n)$
  $\mathrel{\widehat{=}}$ reciprocal of success probability
- result on OneMax: $G_{(1+\lambda)\text{ EA},\text{OneMax}}$ decreases without increasing $T_{(1+\lambda)\text{ EA},\text{OneMax}}$ for
  $\lambda = O(\log(n)\log(\log(n))/\log(\log(\log(n))))$
  no 'fixed' success probability on OneMax
- seen parallelisation simple and effective $\mathrel{\widehat{=}}$ linear speed-up
- Can parallelisation achieve even more?

$(1+\lambda)$ EA and $(1+1)$ EA

Observation $(1+\lambda)$ EA with $\lambda > 1$ is less fast in adopting and exploiting improvements in comparison to $(1+1)$ EA

Can this wastefulness be beneficial even without parallelisation?

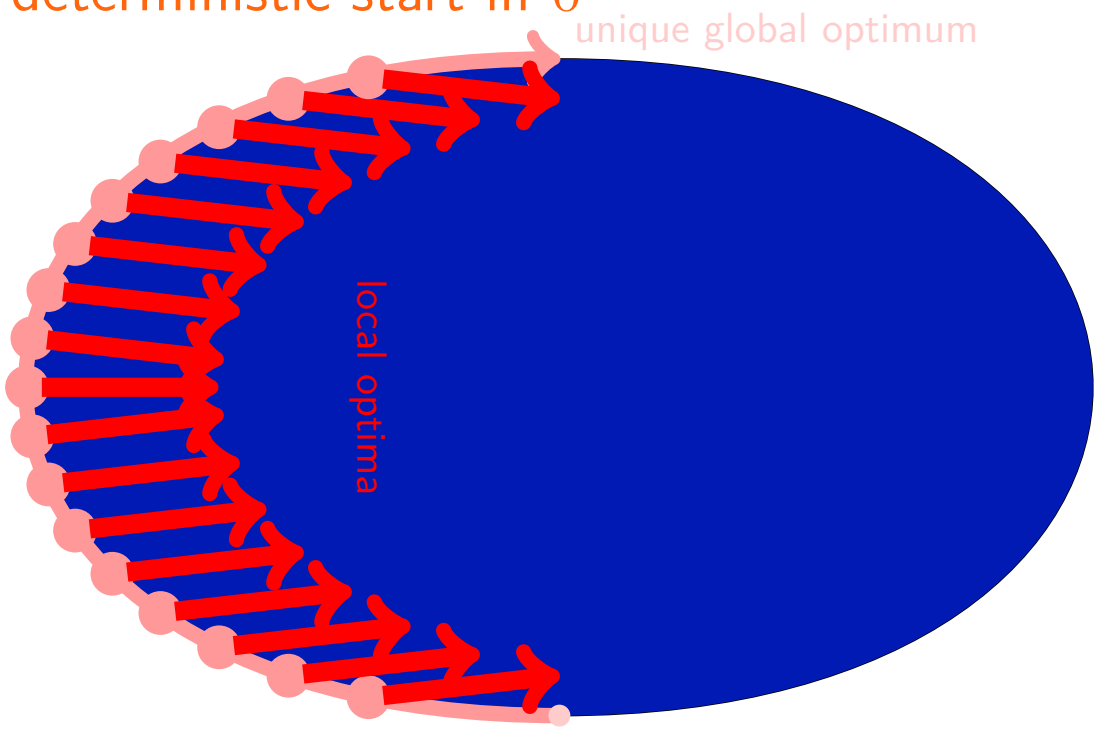
Idea may be beneficial if some improvements are better than others necessary function values give correct hints

An Example Function

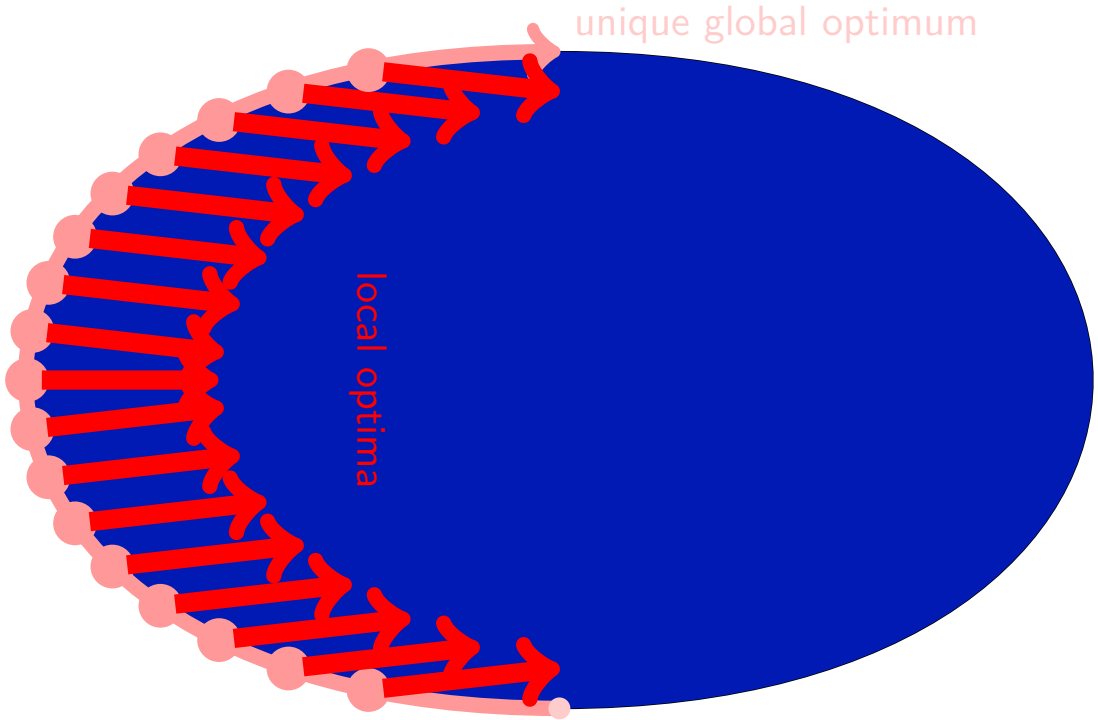
Definition with $k := \lfloor \sqrt{n} \rfloor$

$$f(x) = \begin{cases} (i+3)n + |x| & \text{if } (x = 0^{n-i}1^i \text{ with } i \in \{0,1,\ldots,n\}) \text{ or} \\ & x = y0^{n-i-k}1^i \text{ with } y \in \{0,1\}^k \text{ and} \\ & i \in \{k, 2k, \ldots, (k-2)k\} \\ 0 & \text{otherwise} \end{cases}$$

suitable for deterministic start in 0^n



About the Example Function



Facts

- can be imbedded into a complete function g
- (1+1) EA on g very inefficient (exponential time with probability very close to 1)
- (1+λ) EA with $\lambda \geq n \log n$ efficient, expected time $O(n\lambda)$

Remark roles of local and global optima can be reversed
$\Rightarrow$ reverse performance characteristics

The (1+1) EA

Remember

(1+1) EA

1. $t := 1$; Choose $x_t \in \{0,1\}^n$ uniformly at random.
2. Repeat
3. $\quad y :=$ standard bit mutation(x_t) with mutation prob. p_m
4. $\quad t := t+1$; If $f(y) \geq f(x_{t-1})$ then $x_t := y$ else $x_t := x_{t-1}$.
5. Until 'decide to stop'
6. Output x_t.

Parameter mutation probability $p_m \in (0, 1/2]$, usually $p_m = 1/n$

Remember $\mathrm{E}\,(\#\text{flipping bits}) = p_m \cdot n = 1$ with $p_m = 1/n$

Why should $p_m = 1/n$ be a good universal choice?

About a Universally Optimal Mutation Probability

Why should $p_m = 1/n$ be a good universal choice?

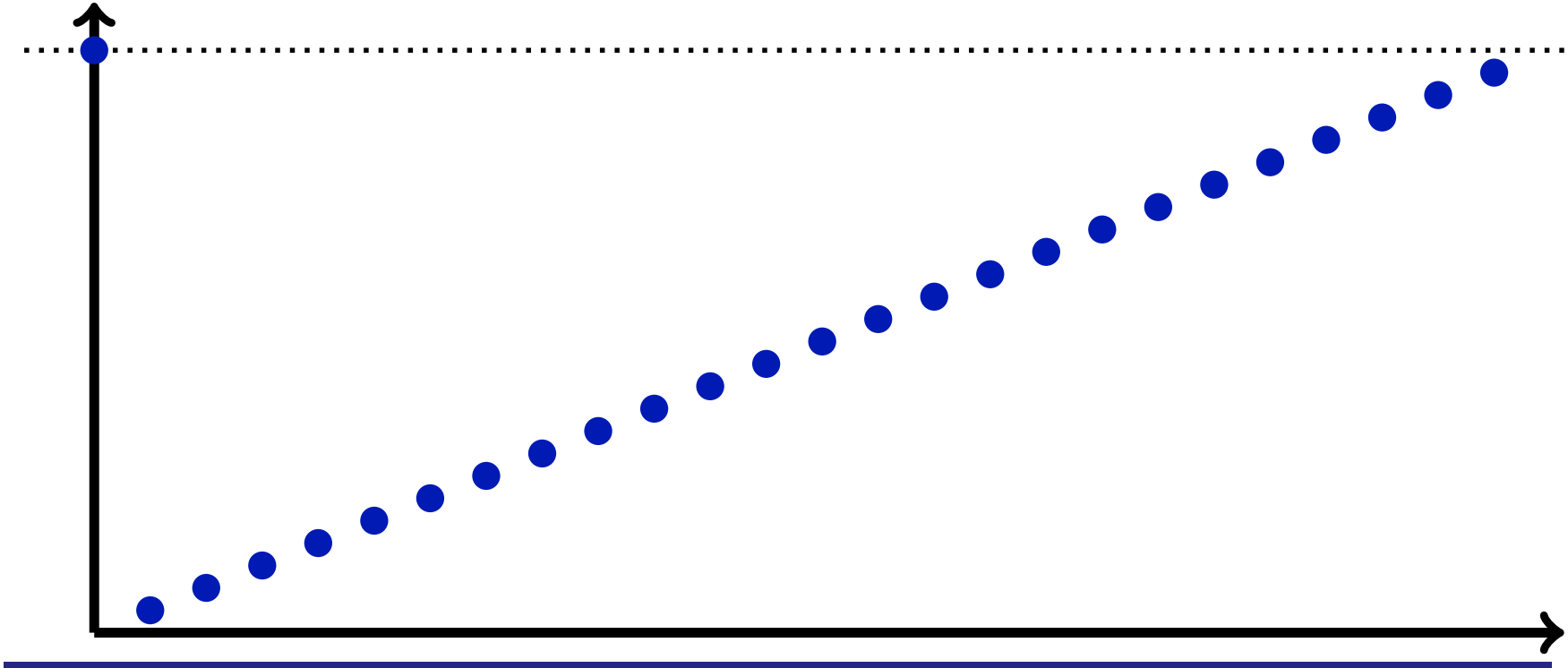
Remember NFL 'Averaged over all functions als RSH perform equal.'
$\widehat{=}$ There is no universally best RSH.
$\widehat{=}$ There is no universally best p_m.

Careful! NFL consider non-repeating algorithms
not actual optimisation times.

Observation for 'normal' RSH not too important
still: Can we find a simple example to prove the point?

A Simple Example

$$f_1(x) = \begin{cases} \textsc{OneMax}(x) & \text{if } x \neq 0^n \\ n+1 & \text{otherwise} \end{cases}$$



Theorem

$\mathrm{E}\left(T_{(1+1)\text{ EA},f_1}\right) = \Theta(n^n)$ with $p_m = 1/n$

$\mathrm{E}\left(T_{(1+1)\text{ EA},f_1}\right) = \Theta(2^n)$ with $p_m = 1/2$

Analysis for the Example Function

$$f_1(x) = \begin{cases} \textsc{OneMax}(x) & \text{if } x \neq 0^n \\ n+1 & \text{otherwise} \end{cases}$$

(1+1) EA with $p_m = 1/n$

- $\text{Prob}\left(\textsc{OneMax}(x_0) \geq n/3\right) = 1 - 2^{-\Omega(n)}$
- given that only direct mutation can lead to optimum
- $\text{Prob}\left(\text{do this before reaching } 1^n\right) = O\Big((n \log n)/n^{n/3}\Big)$
- $\text{E}\left(\text{time } 1^n \rightsquigarrow 0^n\right) = n^n$
- $\Rightarrow \text{E}\left(T_{\text{(1+1) EA},f_1}\right) = \Theta(n^n)$ with $p_m = 1/n$ ✓

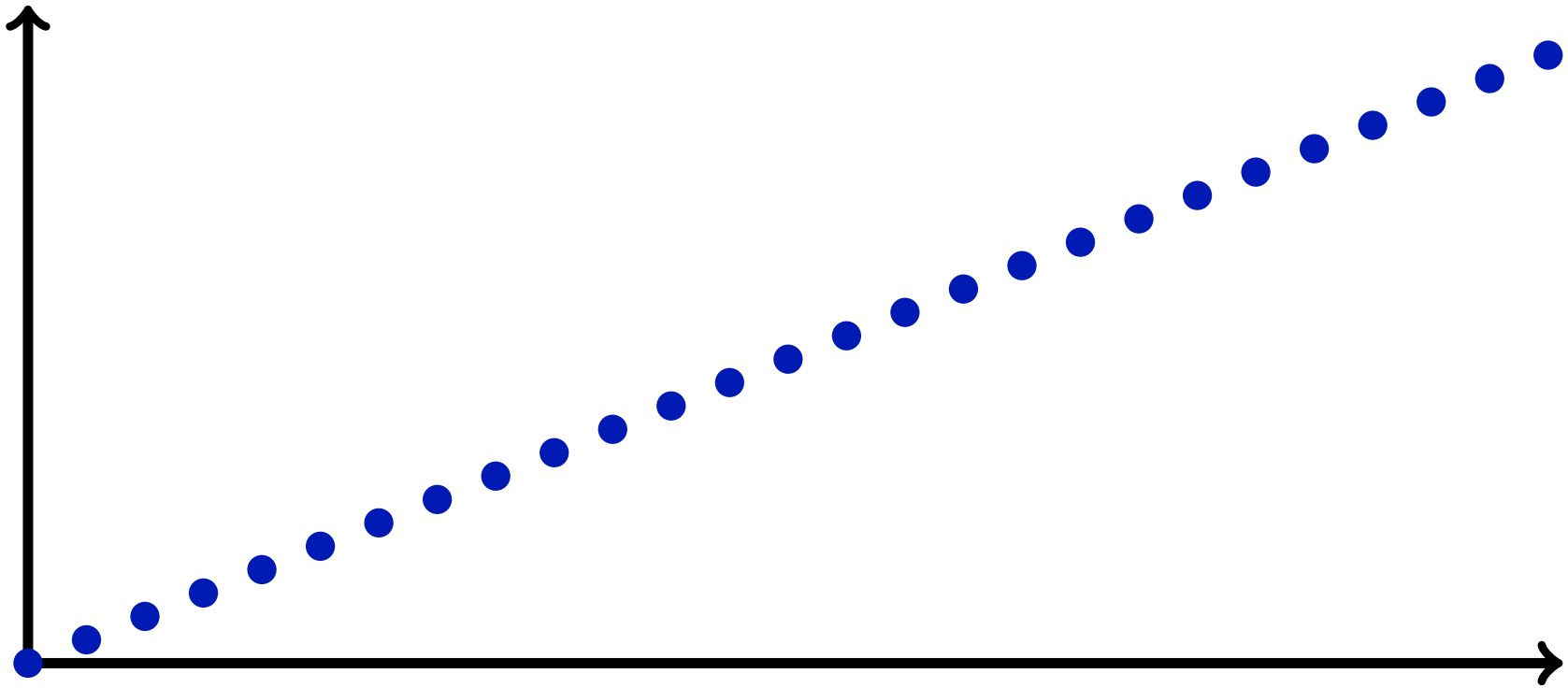
(1+1) EA with $p_m = 1/2$

- $\forall x\colon \text{Prob}\left(x \rightsquigarrow 0^n\right) = 2^{-n}$
- $\Rightarrow \text{E}\left(T_{\text{(1+1) EA},f_1}\right) = \Theta(2^n)$ with $p_m = 1/2$ ✓

□

OneMax

$$\textsc{OneMax}(x) = \sum_{i=1}^{n} x[i]$$



Goal derive simple upper and lower bounds on $\mathrm{E}\left(T_{(1+1)\text{ EA},\textsc{OneMax}}\right)$ depending on p_m

Summary & Take Home Message

Things to remember

- $(1+\lambda)$ EA on OneMax
- super-linear speed-up by parallelisation of EAs
- ideas and example functions
- mutation probability crucial

Take Home Message

- Linear speed-ups only work up to some cut off point that depends on the objective function.
- More extreme positive and negative effects can occur.