Coláiste na hOllscoile Corcaigh, Éire
University College Cork, Ireland

CS4618 Artificial Intelligence I

Today: Crossover
Parametrisation:
Offspring Population Size

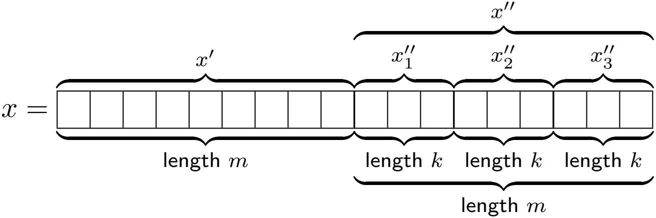
Thomas Jansen

November 28th

Plans for Today

Preparing an Example Function for Uniform Crossover

Notation partition bit string $x = x'x'' = x'x''_1x''_2x''_3$



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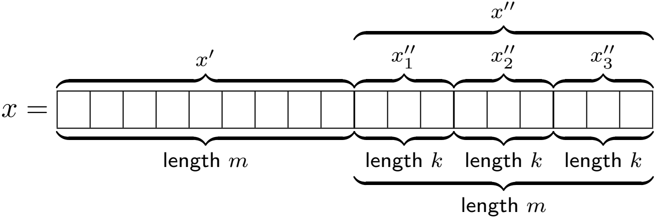

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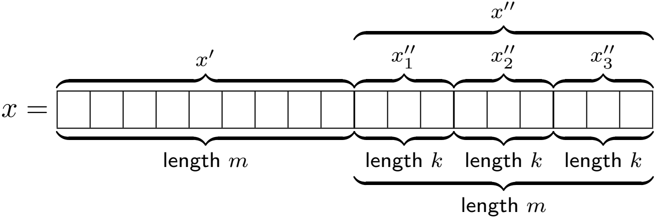

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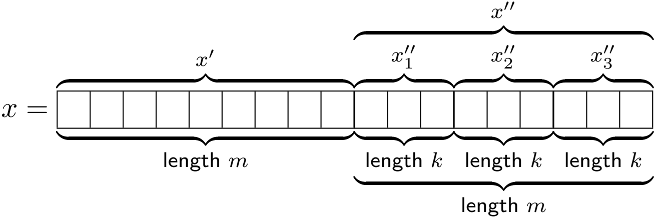

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Definition $f_2\colon \{0,1\}^n \to \mathbb{N}_0$ with

$$f_2(x) = \begin{cases} n - \mathsf{H}(x'', C) & \text{if } x' \neq 0^m \text{ and } x'' \notin C \\ 2n - \mathsf{H}(x', 0^m) & \text{if } x'' \in C \\ 0 & \text{if } x' = 0^m \text{ and } x'' \notin (C \cup T) \\ 3n & \text{if } x' = 0^m \text{ and } x'' \in T \end{cases}$$

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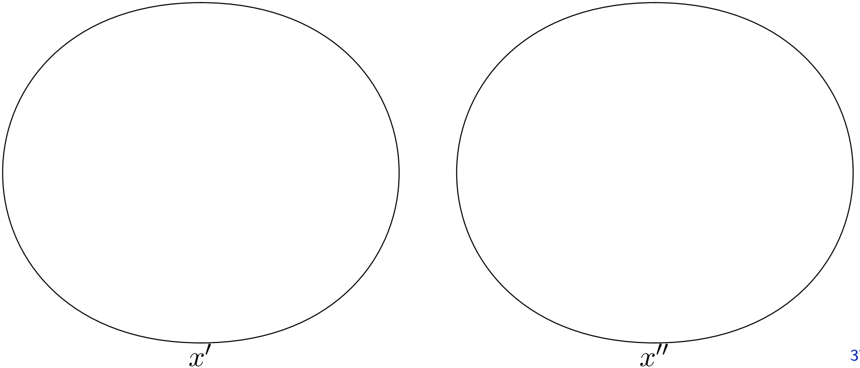

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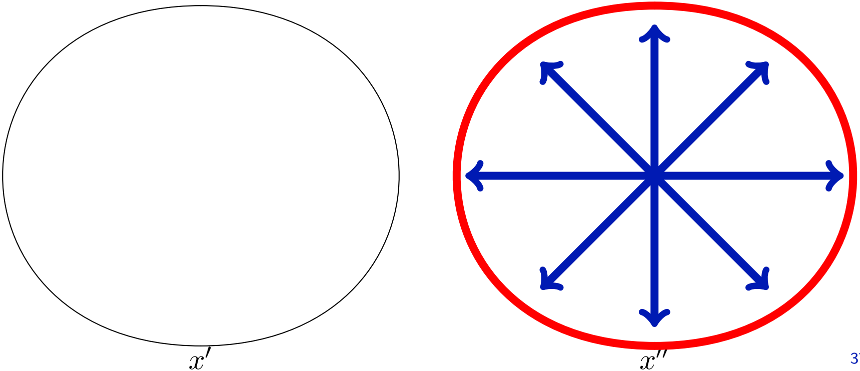

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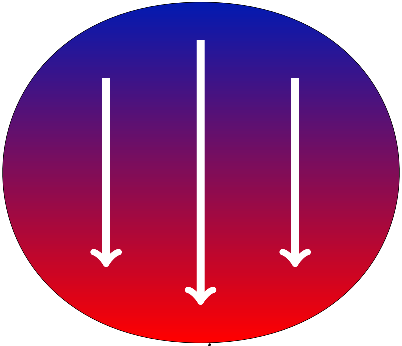

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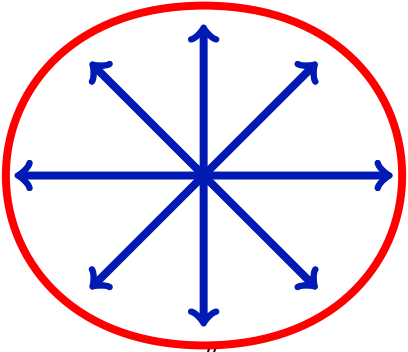

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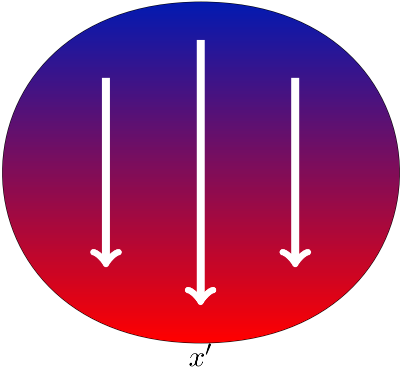

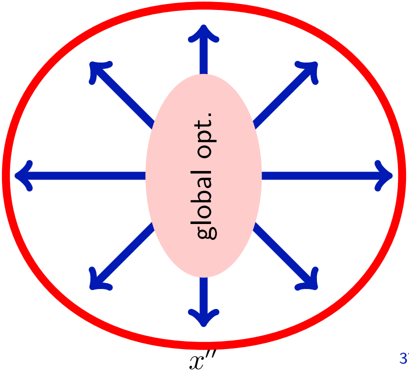

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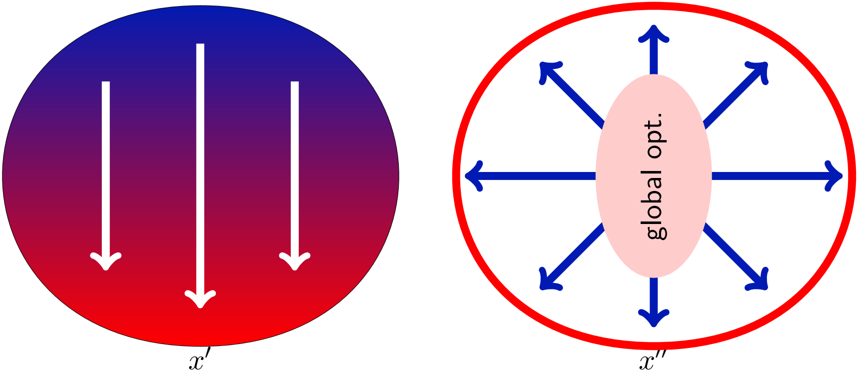

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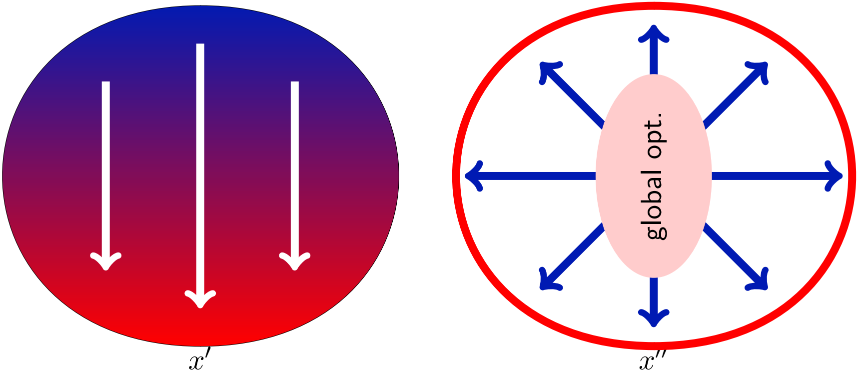

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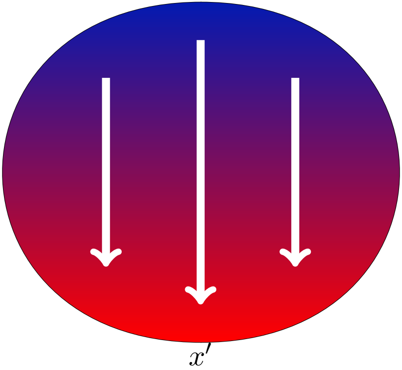

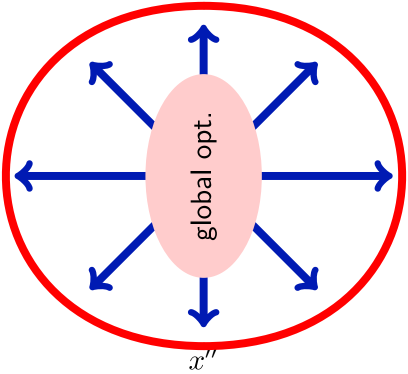

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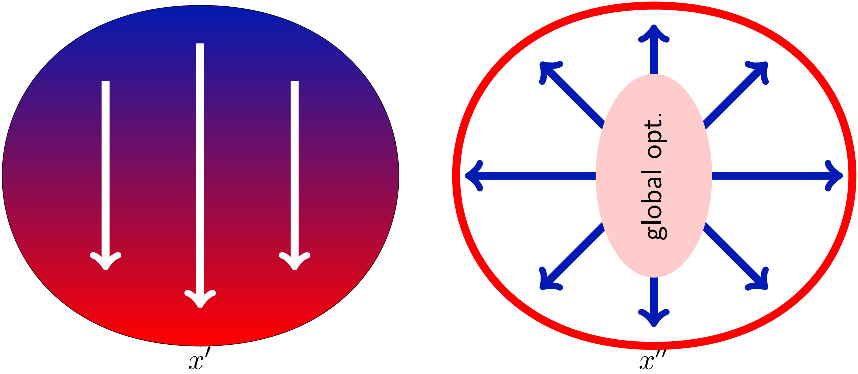

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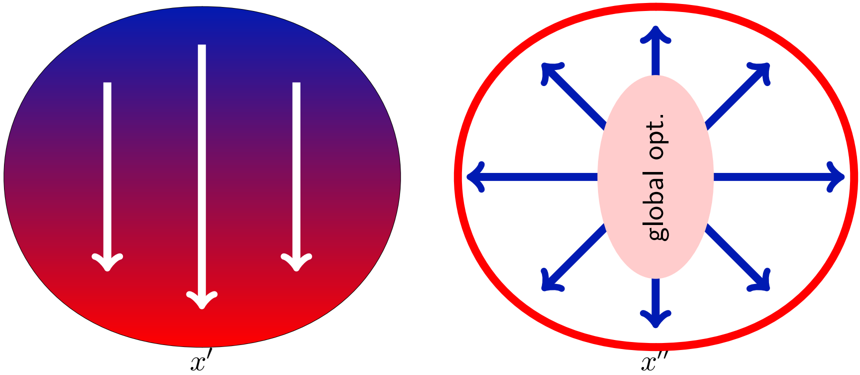
- $\Rightarrow$ exponentially long with overwhelming probability

With Uniform Crossover on f_1



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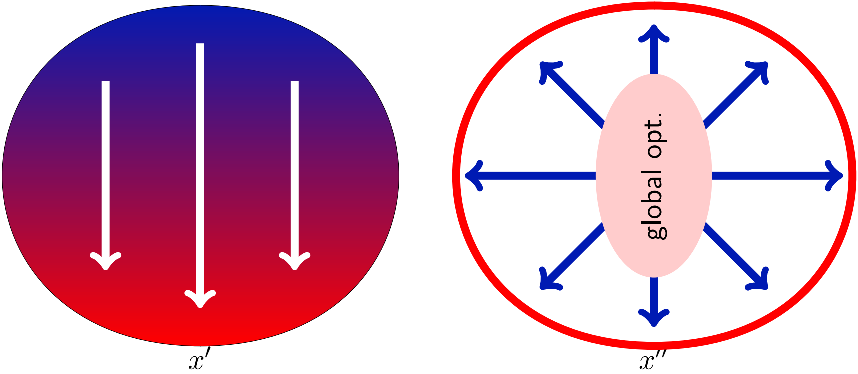

Theorem

Consider the steady state GA with crossover probability $p_c \leq 1 - \varepsilon$ (constant $\varepsilon > 0$) and population size $\mu \geq n$.
$\mathrm{E}\left(T_{\mathsf{GA},f_2}\right) = O\left(\mu n^2 + \mu^2 \log(n)/p_c\right)$

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Observations

- to $x' = 0^m$ and $x'' \in C$ with all possible points on circle in expecation in $O(\mu n^2)$ steps similar to f_1
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- probability to create the optimum $\geq \mathrm{Prob}\left(\text{pick good parents, crossover and generate opt.}\right) \geq (1/\mu^2) \cdot p_c \cdot \mathrm{Prob}\left(\text{generate optimum from } 1^0 0^{m-i} \text{ and } 0^i 1^{m-i}\right)$

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Evolutionary Algorithms

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Remember

Evolutionary Algorithm

1. Choose $x_1, x_2, \ldots, x_\mu \in S$ uniformly at random.
2. Repeat
3. For $i \in \{1, 2, \ldots, \lambda\}$ do
4. With probability p_c select $z_1, z_2 \in \{x_1, x_2, \ldots, x_\mu\}$.
   $z :=$ crossover(z_1, z_2)
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5. $y_i :=$ mutation(z)
6. Select new $x_1, x_2, \ldots, x_\mu$ out of old $x_1, x_2, \ldots, x_\mu$ and $y_1, y_2, \ldots, y_\lambda$.
7 Until 'decide to stop'
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Design Decisions

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Design Choices

- selection for reproduction
- mutation operator
- crossover operator
- selection for replacement

Parameters

- population size $\mu \in \mathbb{N}$
- offspring population size $\lambda \in \mathbb{N}$
- crossover probability $p_c \in [0, 1]$

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Method embed in minimal algorithmic framework

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Method embed in minimal algorithmic framework
consider effects on paradigmatic example functions

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5. $m := \max\{f(y_1), f(y_2), \ldots, f(y_\lambda)\}$
6. If $m \geq f(x)$ Then
   Select $y \in \{y' \in \{y_1, y_2, \ldots, y_\lambda\} \mid f(y') = m\}$ u. a. r.
   $x_{t+1} := y$
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Only Parameter offspring population size λ

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number of generations

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Why should we care about $G_{(1+\lambda)\ \mathsf{EA},f}$?

Observation $G_{(1+\lambda)\ \mathsf{EA},f}$ measures parallel computation time
$\widehat{=}$ computation time on parallel computer
with $\geq \lambda$ processing units

A First Example

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Define $\textsc{LeadingOnes}\colon \{0,1\}^n \to \{0,1,\ldots,n\}$ with

$$\textsc{LeadingOnes}(x) = \sum_{i=1}^{n} \prod_{j=1}^{i} x[j]$$

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counts number of leading 1-bits from left to right

Remember all bits right to the leftmost 0-bit uniformly distributed

Because

- initially true
- bits never involved in selection
- standard bit mutations don't change the uniform distribution

Result

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Theorem

$T_{\text{(1+}\lambda\text{) EA},\textsc{LeadingOnes}} = O\left(n^2 + \lambda \cdot \frac{n}{\log(1+\lambda/n)}\right)$ for any $\lambda = n^{O(1)}$

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Useful Tool Additive Drift Theorem

algorithm A, Z set of all populations,

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Drift Analysis of the (1+λ) EA on LeadingOnes

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Set of Populations $Z =$

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$$\begin{aligned} \mathrm{E}\left(d(x_0)\right) &= \sum_{i=0}^{n} i \cdot \mathrm{Prob}\left(d(x_0) = i\right) \\ &= \sum_{i=1}^{n} i \cdot 2^{-(n+1-i)} \end{aligned}$$

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&= 2^{-(n+1)} \sum_{i=1}^{n} 2^{n+1} - 1 - (2^{i} - 1) \\
&= 2^{-(n+1)} \left(n \cdot 2^{n+1} - \sum_{i=1}^{n} 2^{i} \right)
\end{aligned}
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&= 2^{-(n+1)} \left(n \cdot 2^{n+1} - \sum_{i=1}^{n} 2^{i} \right) \\
&= 2^{-(n+1)} \left(n \cdot 2^{n+1} - 2^{n+1} + 2 \right)
\end{aligned}$$

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&= 2^{-(n+1)} \left(n \cdot 2^{n+1} - 2^{n+1} + 2\right) = n - 1 + 2^{-n}
\end{aligned}
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Drift Analysis of the $(1+\lambda)$ EA on LEADINGONES

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$$= \sum_{i=1}^{d(x_{t-1})} i \cdot \mathrm{Prob}\left(d(x_t) = d(x_{t-1}) - i\right)$$

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$$\geq$$

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&\geq \sum_{i=1}^{d(x_{t-1})} i \cdot \left(1 - \left(1 - \tfrac{1}{en2^i}\right)^{\lambda}\right)
\end{aligned}$$

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Interlude estimating $\left(1 - \frac{1}{en2^i}\right)^{\lambda}$

Bounding $\left(1 - \frac{1}{en2^i}\right)^\lambda$

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Remember $\left(1 - \frac{1}{en2^i}\right)^\lambda$

Bounding $\left(1 - \frac{1}{en2^i}\right)^\lambda$

Remember $$\left(1 - \frac{1}{en2^i}\right)^\lambda = \left(1 - \frac{1}{en2^i}\right)^{en2^i \cdot \lambda/(en2^i)}$$

Bounding $\left(1 - \frac{1}{en2^i}\right)^\lambda$

Remember $\left(1 - \frac{1}{en2^i}\right)^\lambda = \left(1 - \frac{1}{en2^i}\right)^{en2^i \cdot \lambda/(en2^i)} \approx e^{-\lambda/(en2^i)}$

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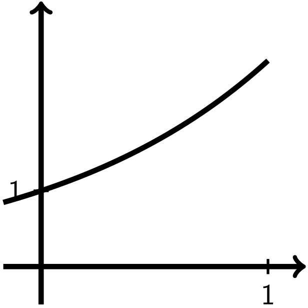
e^x



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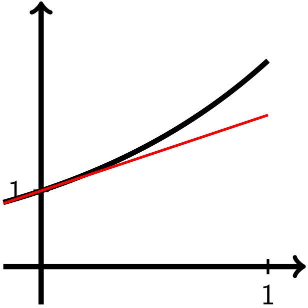
$\approx e^{-\lambda/(en2^i)}$

$\forall x \quad : 1+x \leq e^x$



Bounding $\left(1-\frac{1}{en2^i}\right)^\lambda$

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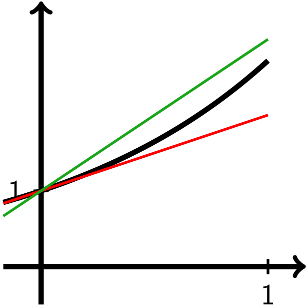

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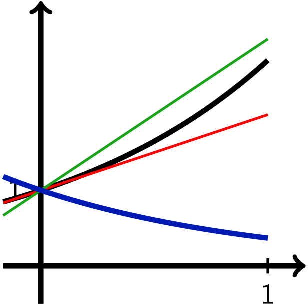
$\forall x \in [0, 1] \colon \qquad e^{-x}$



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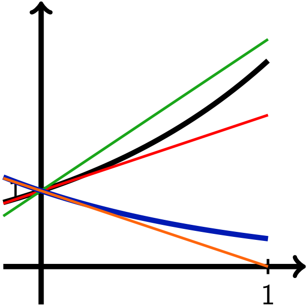

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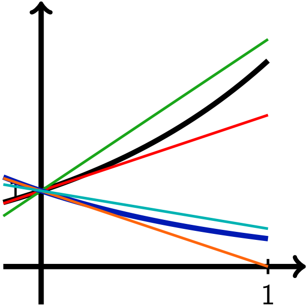

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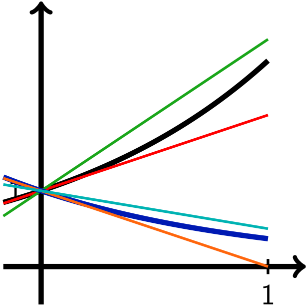

Thus $\left(1-\frac{1}{en2^i}\right)^\lambda \le 1-\frac{\lambda}{2en2^i}$ for $\lambda/(en2^i) \le 1$

Bounding the Drift

Bounding the Drift

We have $$\mathrm{E}\left(D_t \mid P_{t-1}, T_{A,f} \geq t\right) \geq \sum_{i=1}^{d(x_{t-1})} i \cdot \left(1 - \left(1 - \frac{1}{en2^i}\right)^{\lambda}\right)$$

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$$\Rightarrow \mathrm{E}\left(D_t \mid P_{t-1}, T_{A,f} \geq t\right) \geq \log(\lambda/n) \cdot e^{-1/e}$$

Combining Findings

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Remember $\quad \mathrm{E}\left(G_{(1+\lambda)\ \mathsf{EA},\textsc{LeadingOnes}}\right) \leq \mathrm{E}\left(d(x_0)\right)/\Delta^-$

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□

Discussion

Theorem

$$T_{\text{(1+}\lambda\text{) EA},\textsc{LeadingOnes}} = \Theta\left(n^2 + \lambda \cdot \frac{n}{\log(1+\lambda/n)}\right) \text{ for any } \lambda = n^{O(1)}$$

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