Coláiste na hOllscoile Corcaigh, Éire
University College Cork, Ireland

CS4618 Artificial Intelligence I

Today: Analysing Mutation
Applying RSH

Thomas Jansen

November 16th

Plans for Today

Comparing Mutation Operators

Remember (1+1) EA and RLS are algorithmically similar
they only differ in the mutation operator

RLS	1-bit flips
(1+1) EA	flip each bit with probability $1/n$

$\rightsquigarrow$ in expectation 1 bit flipped for both

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  RLS in finite time only with probability 2^{-n+1}
- useful proof method: f-based partitions
- useful visualisation of functions
  - plotted over number of 1-bits (not always possible)
  - as 'map' over search space

On Comparing Mutation Operators

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with global mutations ((1+1) EA)
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More useful question Can global mutations outperform local mutations on functions without local optima?

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Properties

- length $(k+1) \cdot 2^{(n-1)/k} - k + 1$
- $\forall p \in P_k^n \colon \forall i \in \{1, 2, \ldots, k-1\} \colon \exists_1$successor $y \in P_k^n \colon$
  $\mathsf{H}(p, y) = i$

Long Path Function

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Example

$$f_2(x) := \begin{cases} n^2 + p & \text{if } x = v_p \in P_2^n \\ n^2 - n \cdot \left(\sum_{i=1}^{2} x[i]\right) - \sum_{i=3}^{n} x[i] & \text{otherwise} \end{cases}$$

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Easy to prove $\quad \mathrm{E}\left(T_{\mathsf{RLS}, f_2}\right) = \Omega(n \cdot |P_2^n|) = \Omega\Big(n \cdot 2^{n/2}\Big)$

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(formal proof with f_2-based partition) ✔

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Can global mutations be very much slower than local mutations?

Trapping Global Mutations

$$f_3(x) := \begin{cases} n + 2i & \text{if } x = 1^i 0^{n-i} \\ 3n - 1 & \text{if } x = 1^i 0^j 1 0^k 1 0^{n-i-j-k-2} \\ & \text{with } n/4 \leq i \leq 3n/4 \text{ and } j > 0 \\ n - \text{OneMax}(x) & \text{otherwise} \end{cases}$$

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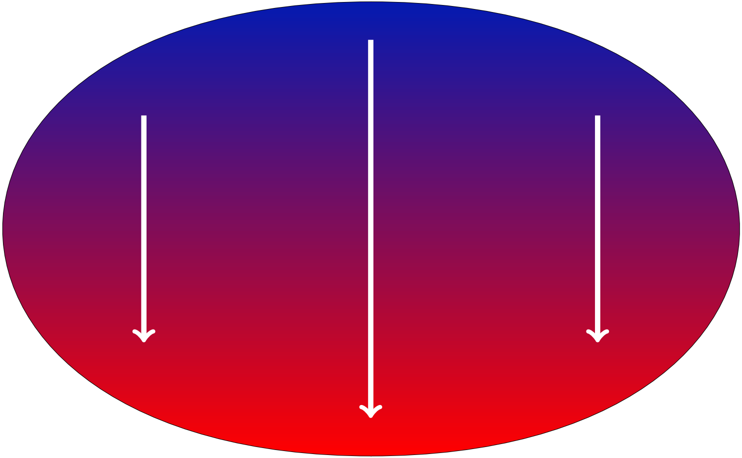

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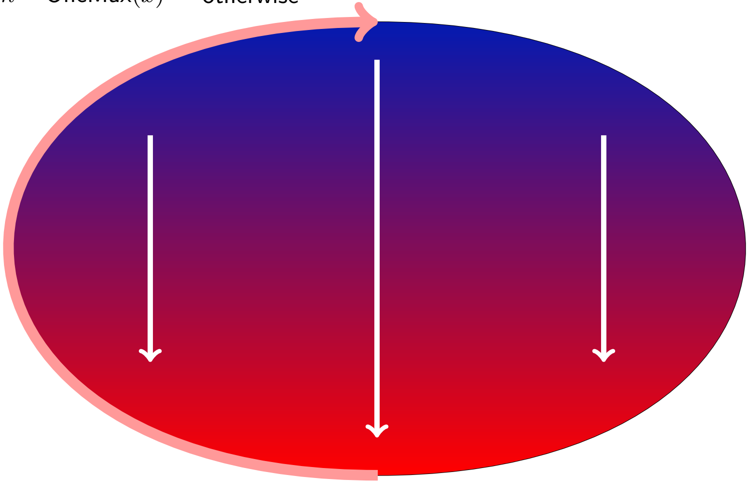

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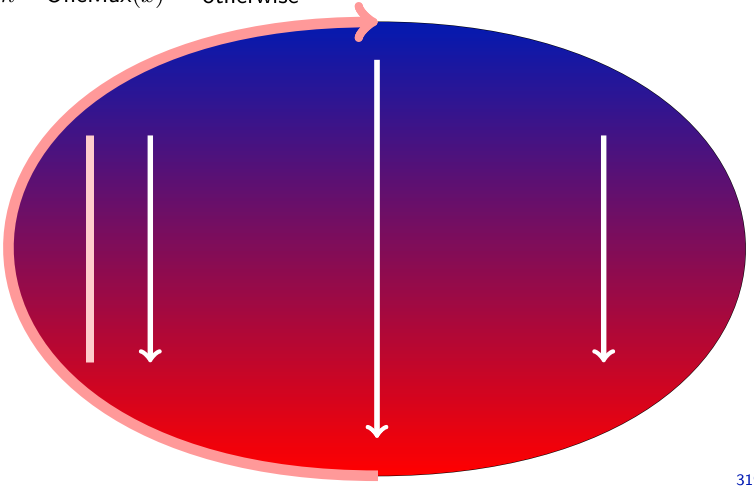

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Observation $\text{Prob}\left(\text{trap not found}\right) = 1 - 2^{-\Omega(n)}$ ✔

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$\text{E}\left(T_{\text{RLS},f_3} \mid \text{trap not found}\right) = O(n \log n) + \text{E}\left(\text{time on ridge}\right)$

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Thus $\mathrm{E}\left(T_{\text{RLS}, f_3} \mid \text{trap not found}\right)$
$= O(n \log n) + \mathrm{E}\left(\text{time on ridge}\right)$
$= O(n \log n) + O\left(n^2\right) = O\left(n^2\right)$

□

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Proof

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$$\text{Prob}\left(T_{\text{(1+1) EA}, f_3} \geq n^{n/4}\right) = 1 - 2^{-\Omega(n)}$$

Proof

Observation $\quad \mathrm{E}\left(T_{\text{(1+1) EA}, f_3} \mid \text{trapped}\right) \geq \left(\left(\frac{1}{n}\right)^{n/4}\right)^{-1} = n^{n/4}$

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Thus $\text{Prob}\,(\text{don't enter trap}) = 1 - 2^{-\Omega(n)}$ □

One Word on Research

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When do local and global mutations perform similar?

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Idea when there are no local optima and no traps

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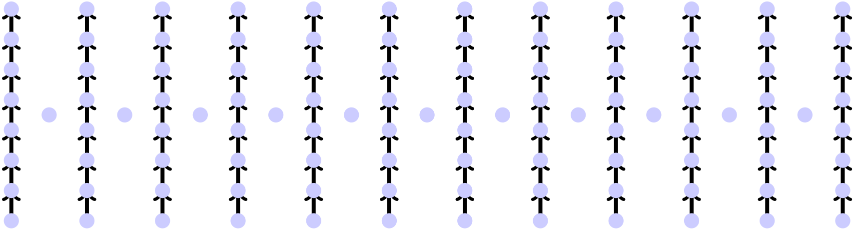

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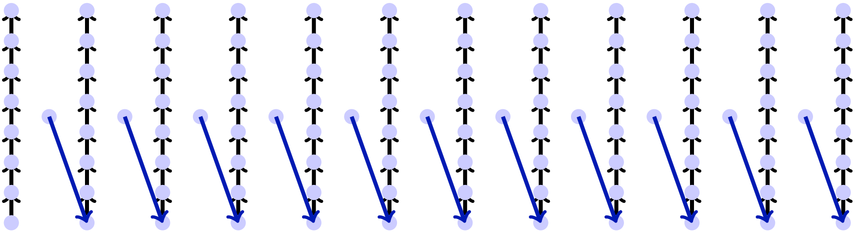

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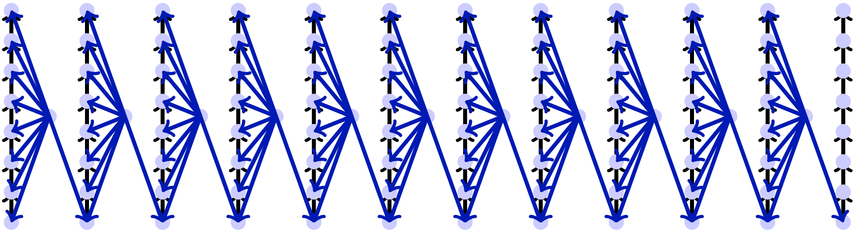

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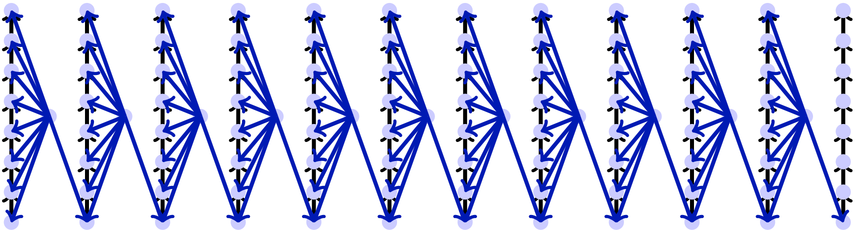

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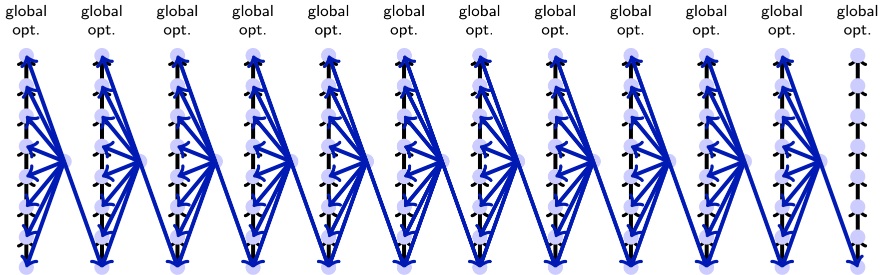

Embedding the Counter-Example in $\{0, 1\}^n$

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Definition

For $n \in \mathbb{N}$ with $n \geq 16$
$n_1 := 4 \cdot \lfloor n/8 \rfloor$, $k := \left\lfloor \sqrt{n_1 - 1} \right\rfloor$, $n_2 := k^2 + 1$, $n_3 := n - n_1 - n_2$.

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$$f_4(x) := \begin{cases} 2^n \cdot n^2 & \text{if } x_1 = 1^{n_1} \text{ or } x_2 = v_l \\ i \cdot n^2 + j & \text{if } x_1 = 1^j 0^{n_1 - j},\ x_2 = v_i \\ i \cdot n^2 + n + n_1 - j_1 & \text{if } x_1 = 1^{j_1} 0^{j_2} 1 0^{j_3} 1 0^{n_1 - j_1 - j_2 - j_3 - 2},\ x_2 = v_i, \\ & n_1/4 \leq j_1 + j_2 \leq n_1/2,\ j_1 + j_2 + j_3 \geq (3/4) n_1 \\ i \cdot n^2 + 2n & \text{if } x_1 = 0^j 1 0^{n_1 - j - 1},\ n_1/4 \leq j \leq n_1/2, \\ & x_2 = v_i,\ i \text{ odd} \\ i \cdot n^2 - 1 & \text{if } x_1 = 0^j 1 0^{n_1 - j - 1},\ n_1/4 \leq j \leq n_1/2, \\ & x_2 = v_i,\ i \text{ even} \\ i \cdot n^2 + 2n & \text{if } x_1 = 0^j 1 0^{n_1 - j - 1},\ j > n_1/2, \\ & x_2 = v_i,\ i \text{ even} \\ i \cdot n^2 - 1 & \text{if } x_1 = 0^j 1 0^{n_1 - j - 1},\ j > n_1/2, \\ & x_2 = v_i,\ i \text{ odd} \\ n - |x| & \text{otherwise} \end{cases}$$

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