ucc
Coláiste na hOllscoile Corcaigh, Éire
University College Cork, Ireland

CS4618 Artificial Intelligence I

Today: Heuristics for A* Search
Adversarial Search ($\widehat{=}$ Games)

Thomas Jansen

October 10th

Plans for Today

Generating Heuristic Functions

Remember A* search
$\widehat{=}$ best-first search guided by evaluation function $f = g + h$
(g = path cost, h = heuristic)

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- constructing heuristics from relaxations

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- constructing heuristics from relaxations
- constructing heuristics from subproblems

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- constructing heuristics from relaxations
- constructing heuristics from subproblems
- learning heuristics from experience

Relaxations

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For a given problem a relaxation of the problem is created by removing restrictions on the actions.

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Example modifications of 8-puzzle

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Example modifications of 8-puzzle

Remember rule for 8-puzzle
action can move tile from A to B if and only if
1. A is adjacent to B and
2. B is empty.

No other tile is moved.

Relaxations of the 8-Puzzle

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- optimal value $\widehat{=}$ number of misplaced tiles
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- Relaxation 1 generates h_1

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- optimal value $\widehat{=}$ sum of Manhattan distances over all tiles

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Observation $\forall v\colon h_1(v) \leq h_3(v)$

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3	7	5
6	4	8

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Remark Relaxation 3 generates Gaschnig's heuristic
J. Gaschnig (1979): Performance Measurement and Analysis of Certain Search Algorithms. PhD Thesis, CMU.

On Having Many Heuristics

Remember for 8-puzzle we have three heuristics

- h_1: number of misplaced tiles
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$h(v) := \max\{h_1(v), h_2(v), \ldots, h_n(v)\}$

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Is h also consistent?

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$\exists v, a\colon h(v) > \text{cost}(a) + h(v')$ with $v' = \text{transition-model}(v, a)$

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- $\exists i, j\colon h(v) = h_i(v), h(v') = h_j(v')$ by definition of h

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- $h_i(v) \leq \text{cost}(a) + h_i(v')$, $h_j(v) \leq \text{cost}(a) + h_j(v')$ since h_i and h_j consistent

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- $i \neq j$ since h_i consistent
- $h_i(v) \geq h_j(v)$, $h_j(v') \geq h_i(v')$ by definition of h
- $h_i(v) \leq \text{cost}(a) + h_i(v')$, $h_j(v) \leq \text{cost}(a) + h_j(v')$ since h_i and h_j consistent
- $h_i(v') > h_j(v')$ since $h_i(v) \leq \text{cost}(a) + h_i(v')$ and $h_i(v) > \text{cost}(a) + h_j(v')$ 

Thus h consistent

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Idee define subproblem ($\approx$ smaller problem)
use cost of solving the subproblem as heuristic

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*		*
*	3	1

and goal state

	1	2
3	4	*
*	*	*

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and goal state

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*	*	*

Idea in preprocessing compute solutions for each subproblem and store in database, used for computing h

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Example for 8-puzzle any initial state like

*	2	4
*		*
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and goal state

	1	2
3	4	*
*	*	*

Idea in preprocessing compute solutions for each subproblem and store in database, used for computing h

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Alternative for 8-puzzle any initial state like

*	8	6
*		*
*	7	5

and goal state

	*	*
*	*	5
6	7	8

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Can we combine these?

Combining 1–4 Subproblem Heuristic with 5–8 Subproblem Heuristic

Remember heuristic based on all 1–4 initial states
heuristic based on all 5–8 initial states

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- correct if only moves of considered tiles are counted

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  $\rightsquigarrow$ very good heuristics

Learning Heuristics

An Outlook on 'self-improving' agents

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Consider agent solving problem with A* search with heuristic $h = 0$

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Improvement use some way of extending knowledge
using values for similar states

Search

Search

We know

- uninformed search
  - BFS
  - DFS

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Now search for a special kind of application 'games'
adversarial search

'Classical' Game Theory

Consider two-player zero-sum games.

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row player aims at maximising.

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mixed $\widehat{=}$ randomised

Set-Up and Notation

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Central two-player zero-sum games
but with emphasis on alternating moves

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Remark relaxation of 'zero-sum game'
$\forall$terminal states s: Utility$(s, \text{Max}) + \text{Utility}(s, \text{Min})$ equal
(not necessarily equal to 0)

Games and Search

Observation

- initial state: S_0 (game set up)
- actions: set of legal moves
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- terminal test for end of game

$\widehat{=}$ given like in a search problem

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Thus game tree
in the same way as search tree

Game Tree

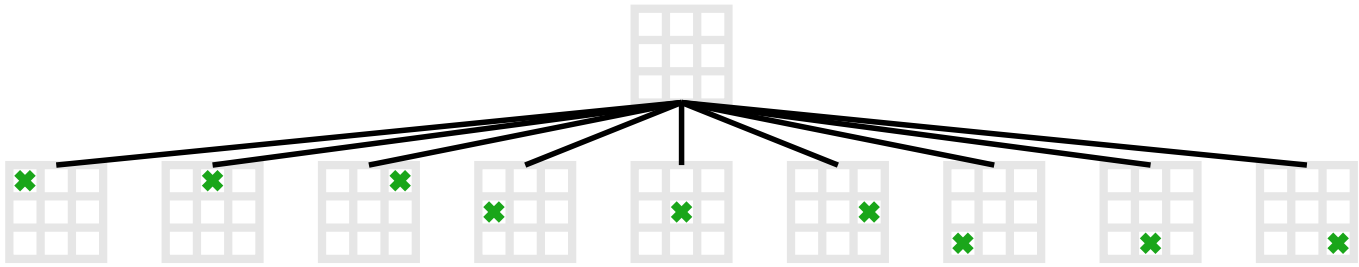
Game Tree

Max

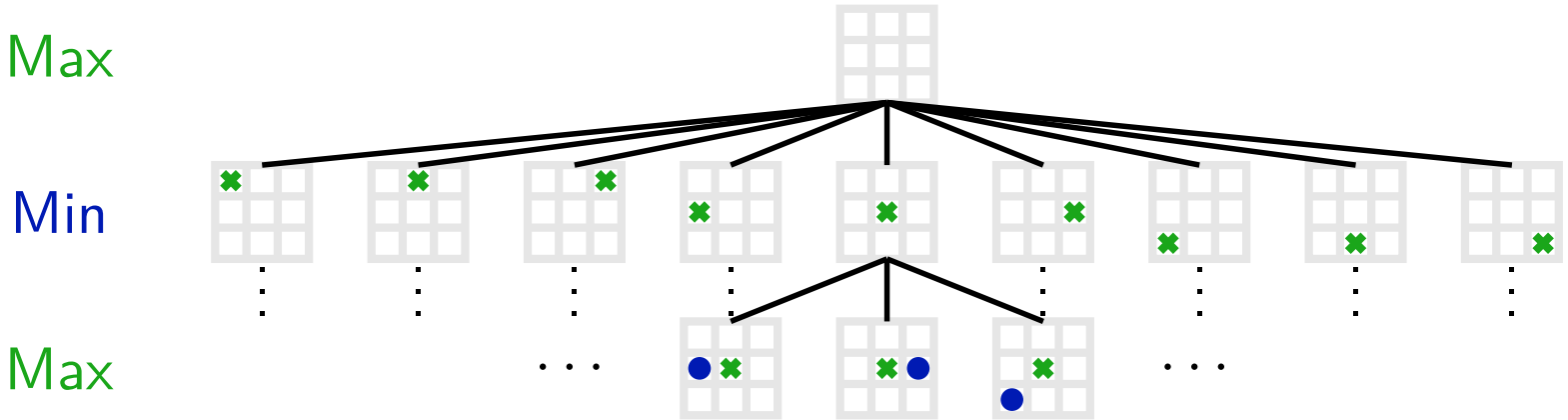
Game Tree

Max

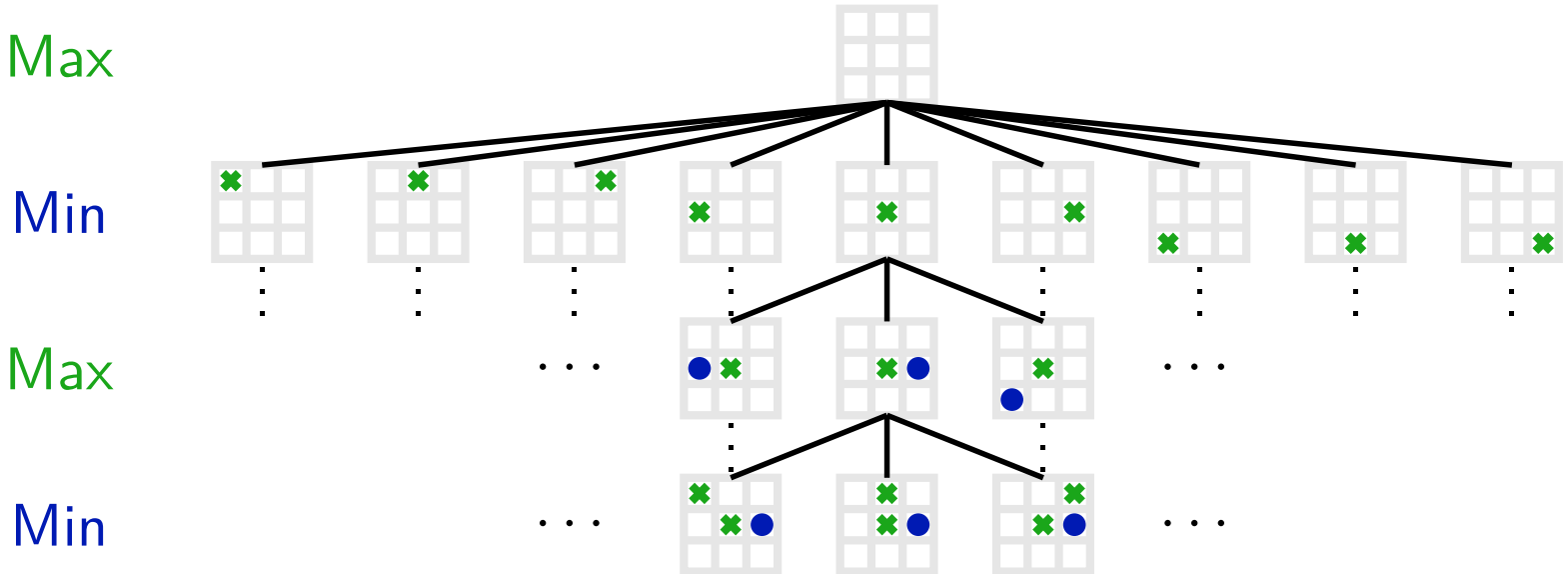
Min



Game Tree



Game Tree



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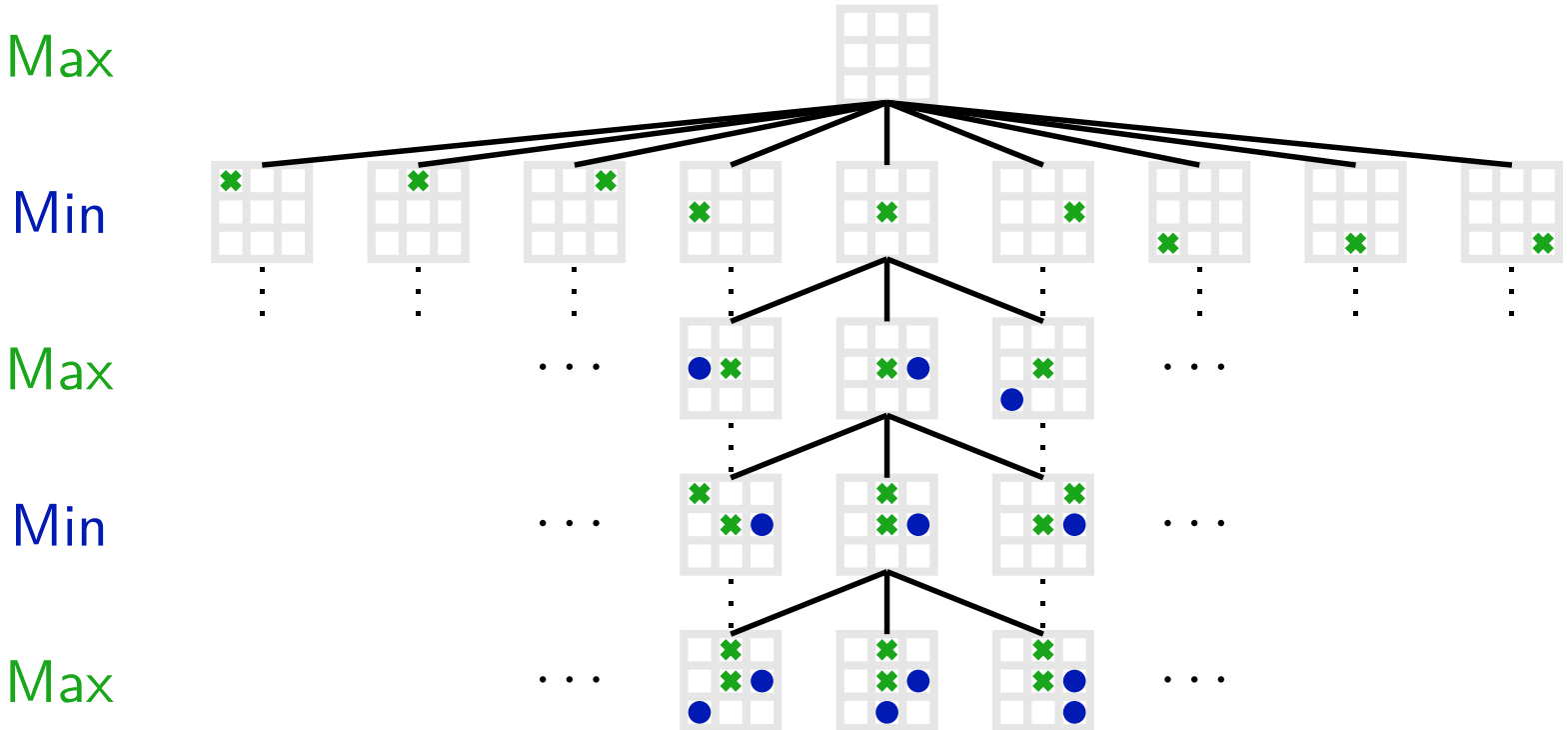

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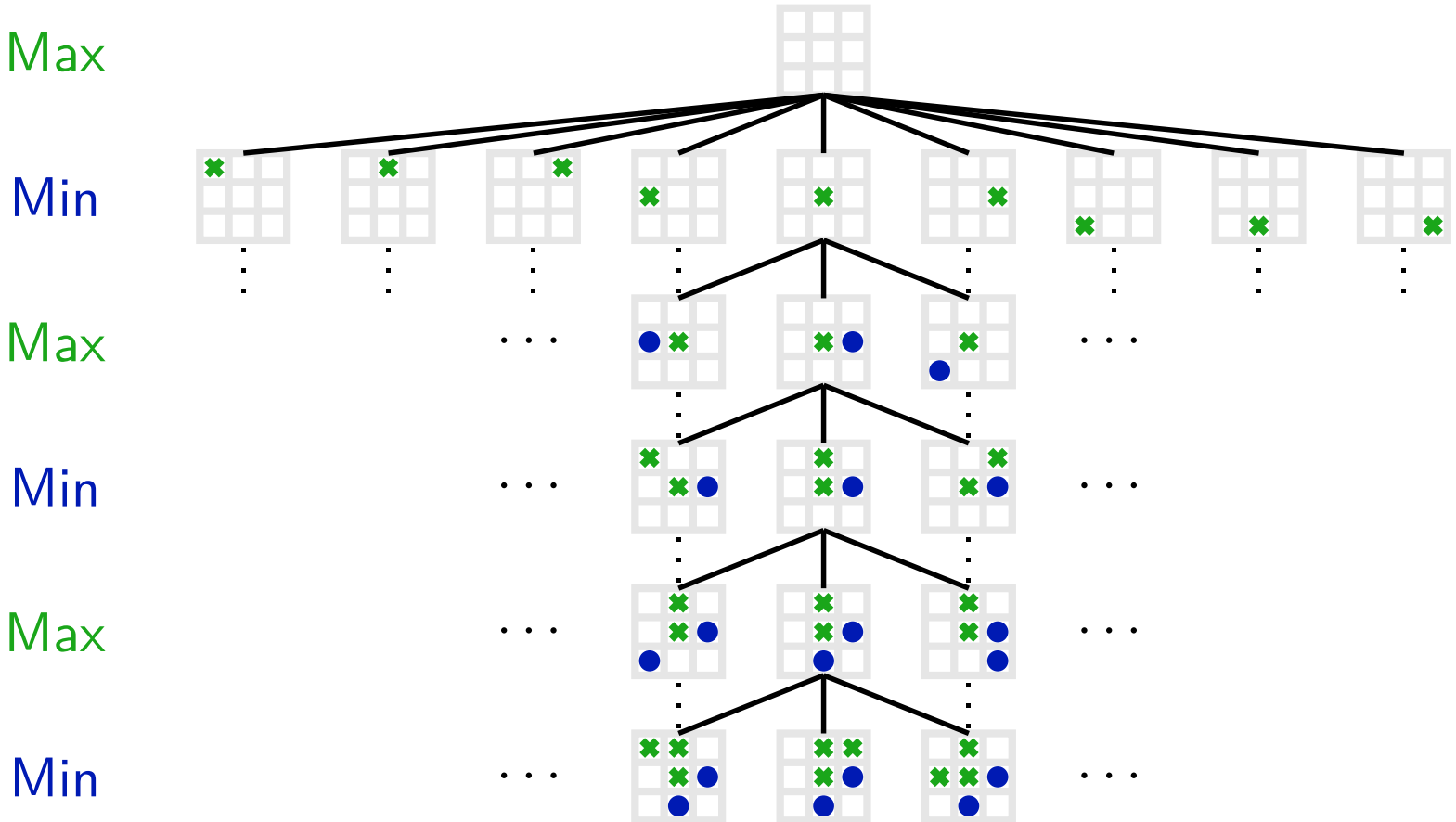

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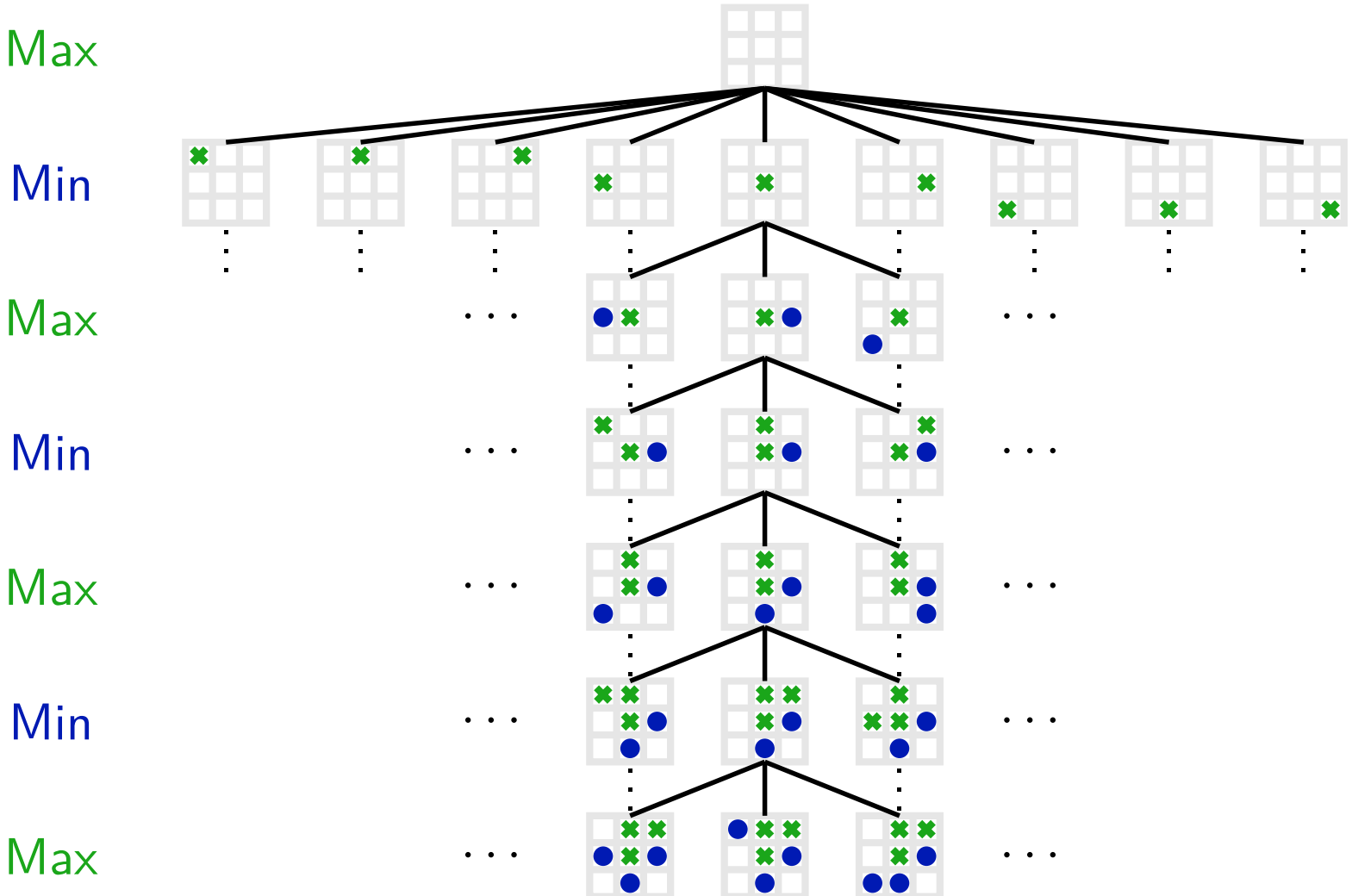

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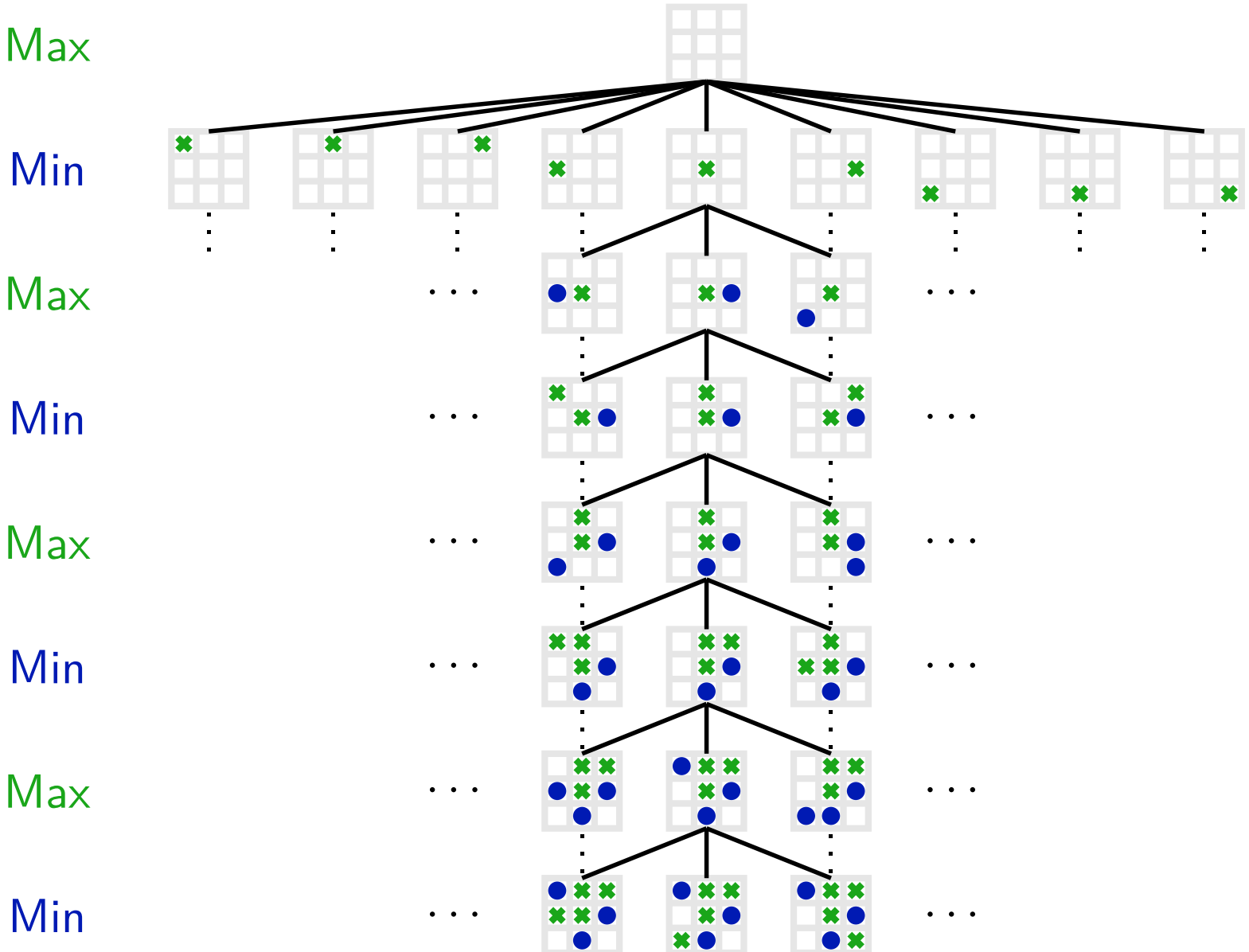

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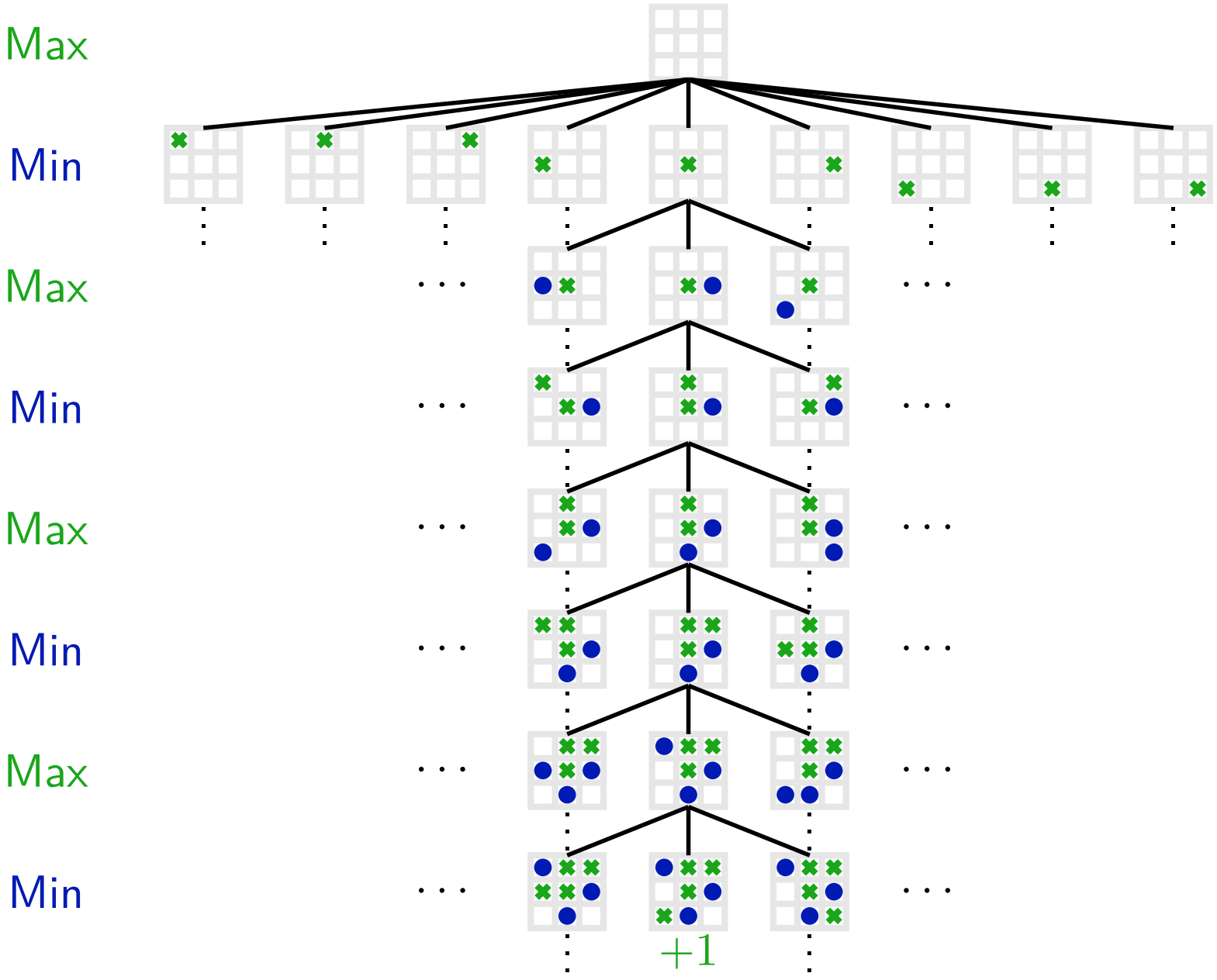

Minimax Search

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Remember Max wants to maximise
Min wants to minimise
the final outcome

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Definition for an arbitrary state s

$$\text{minimax}(s) = \begin{cases} \text{utility}(s) & \text{if terminal-test}(s) \\ \max_{a \in \text{actions}(s)} \text{minimax}(\text{result}(s,a)) & \text{if player}(s) = \text{Max} \\ \min_{a \in \text{actions}(s)} \text{minimax}(\text{result}(s,a)) & \text{if player}(s) = \text{Min} \end{cases}$$

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Example Take(n,k)
Start with n. $(s_0 = n)$

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Start with n. $(s_0 = n)$
Each player takes $i \in \{1, 2, \ldots, k\}$
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First player to reach 0 wins.

Take$(5, 3)$

Take$(5, 3)$

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Take$(5, 3)$

Max

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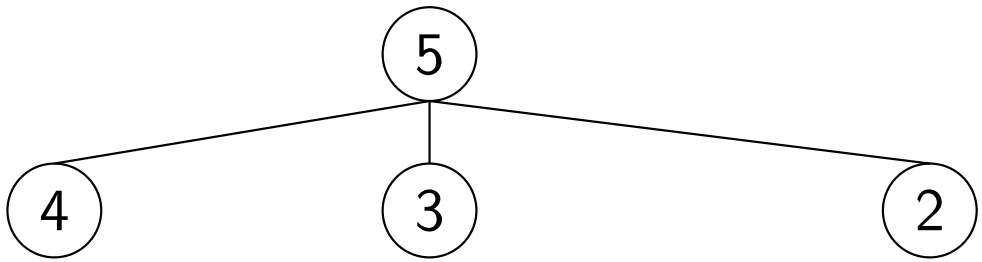

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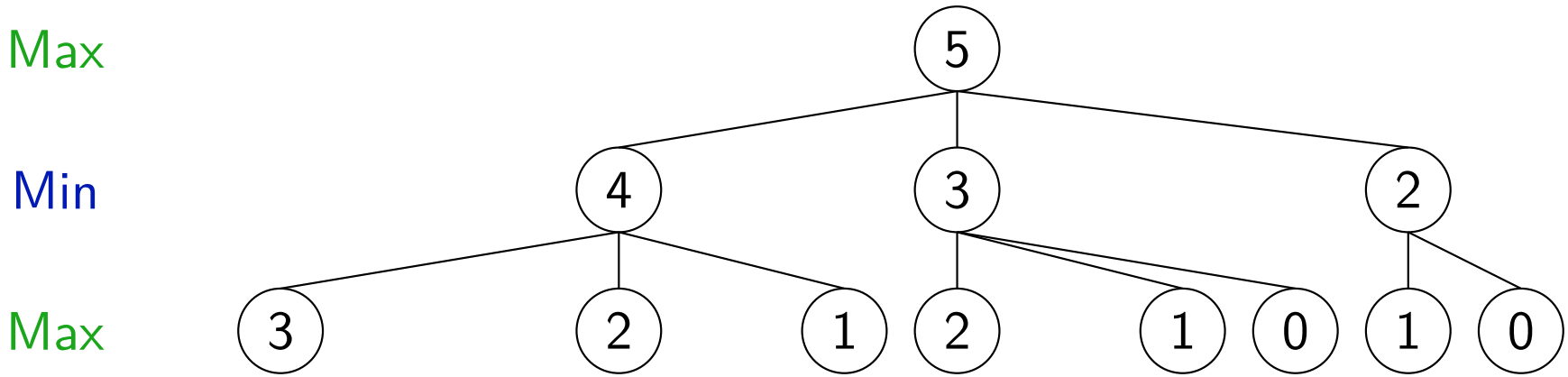

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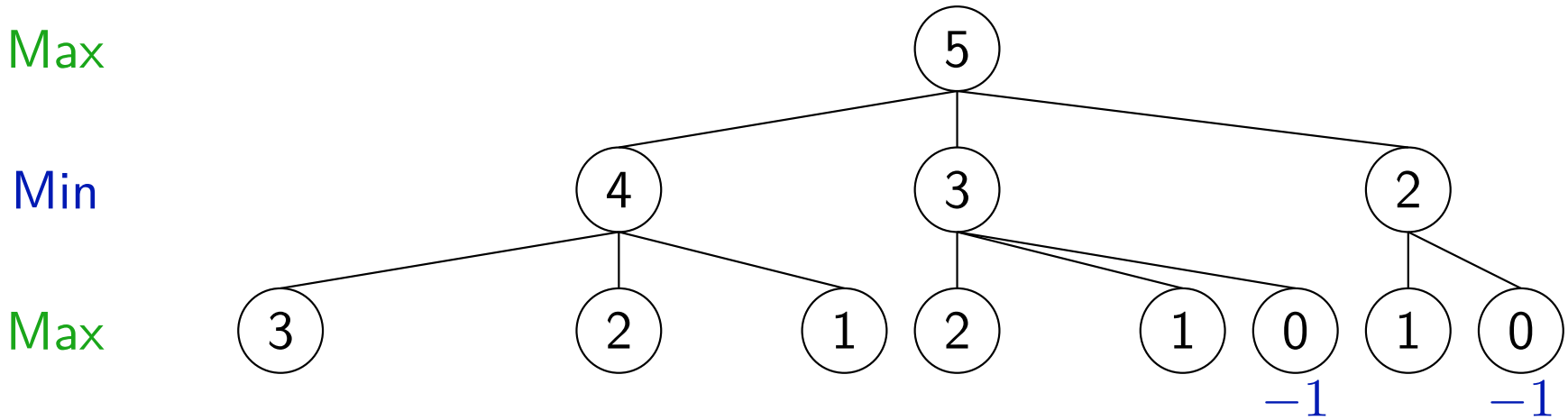

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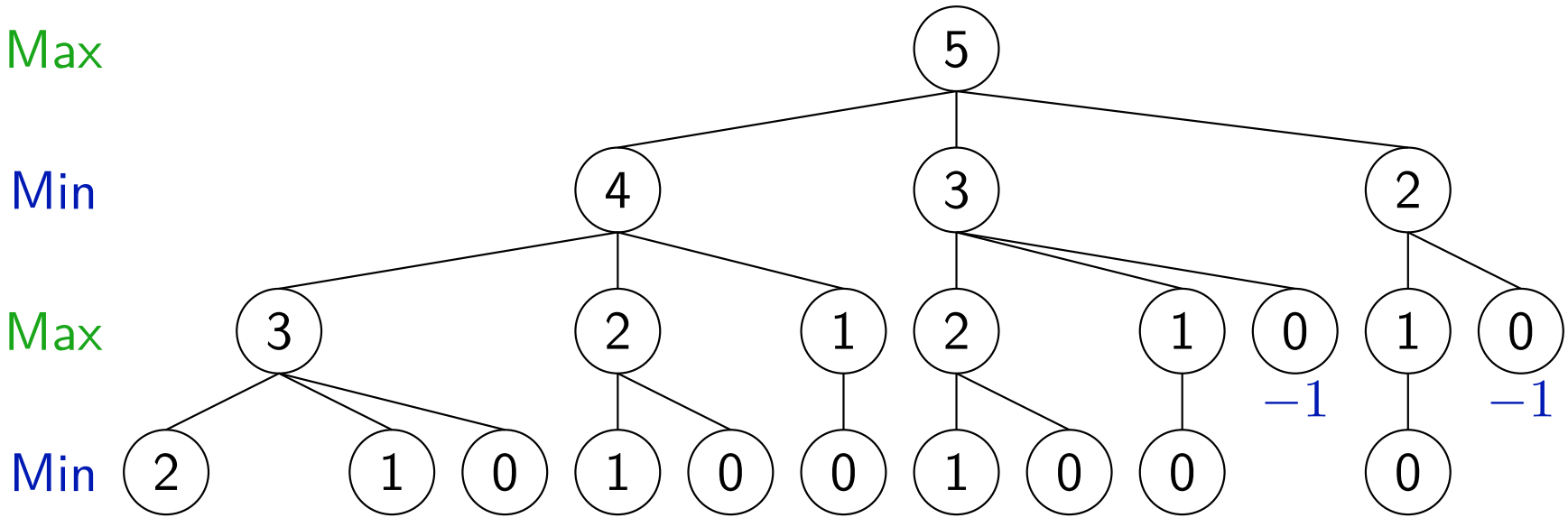

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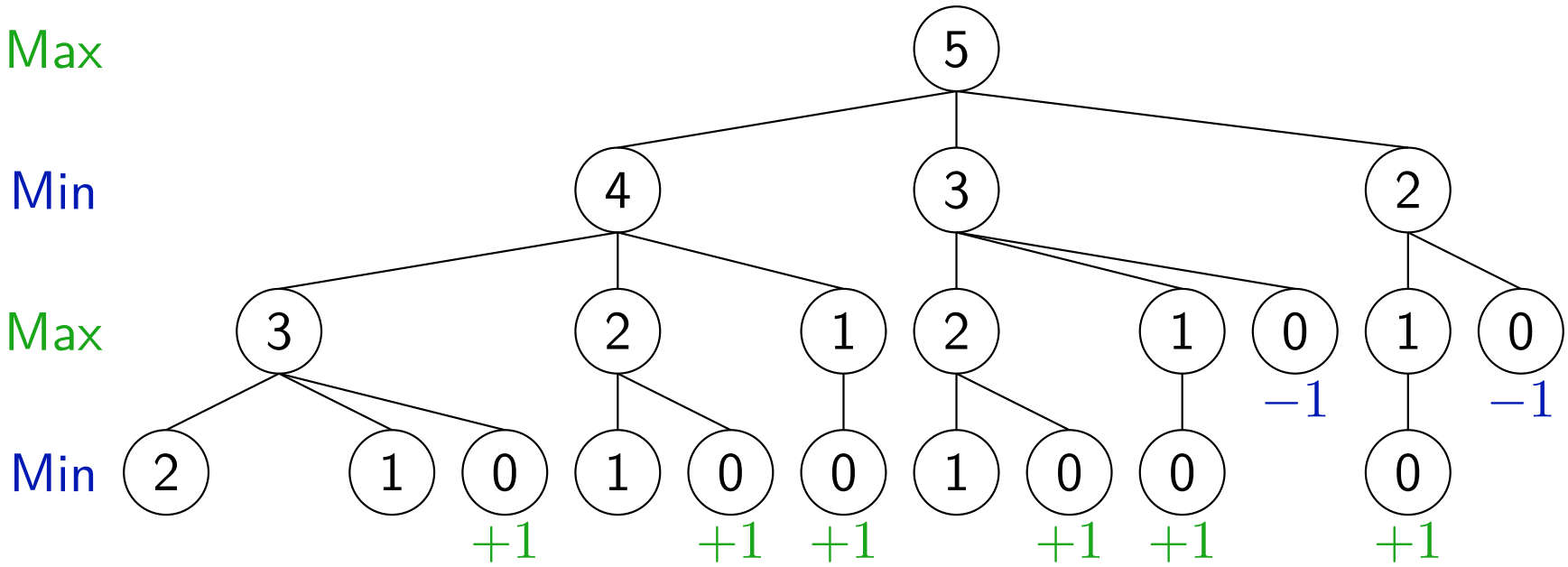

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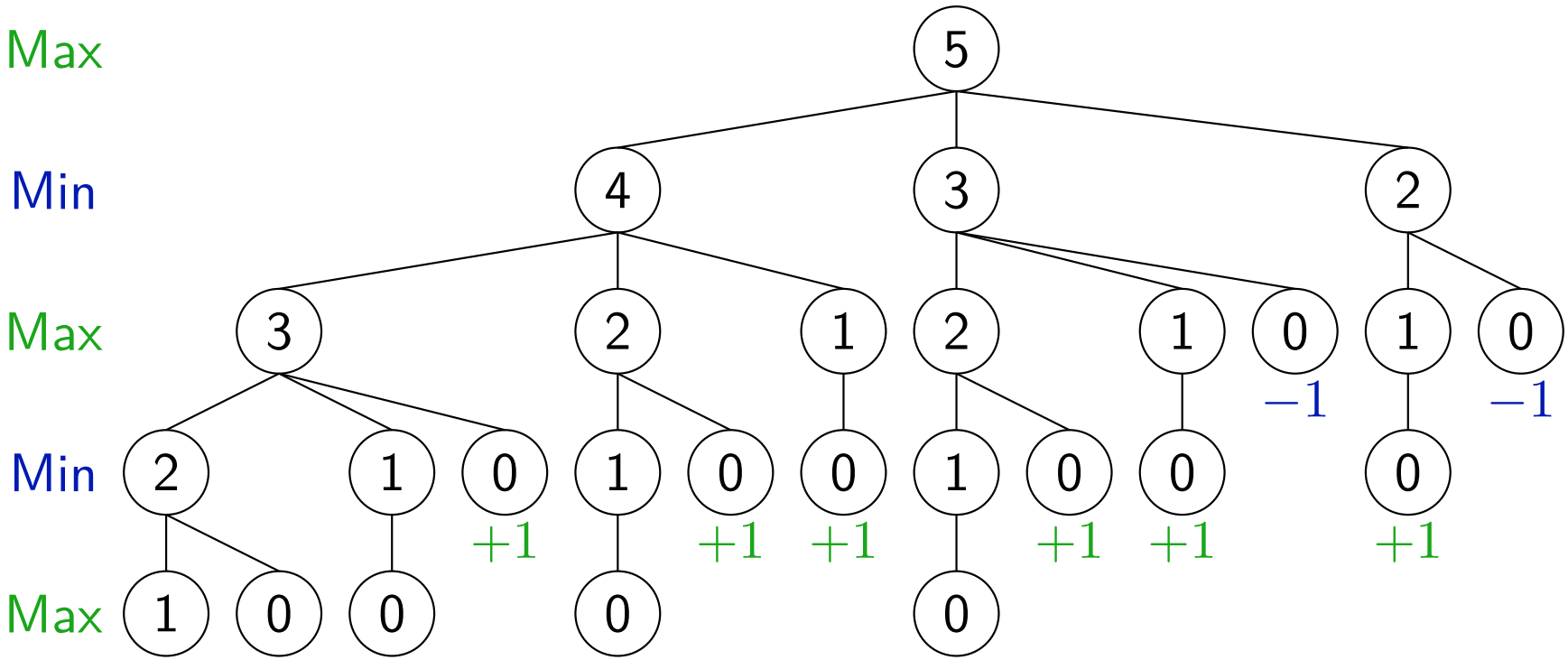

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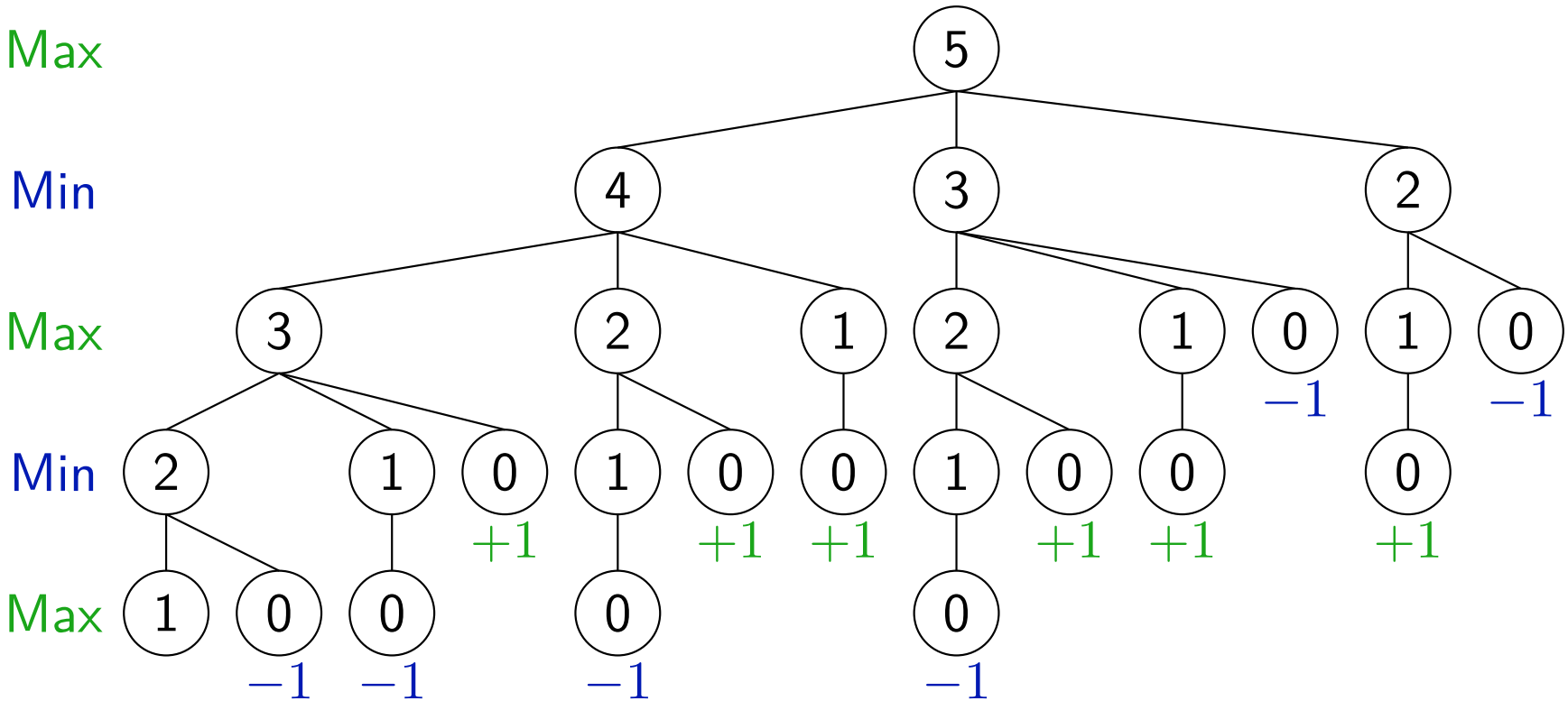
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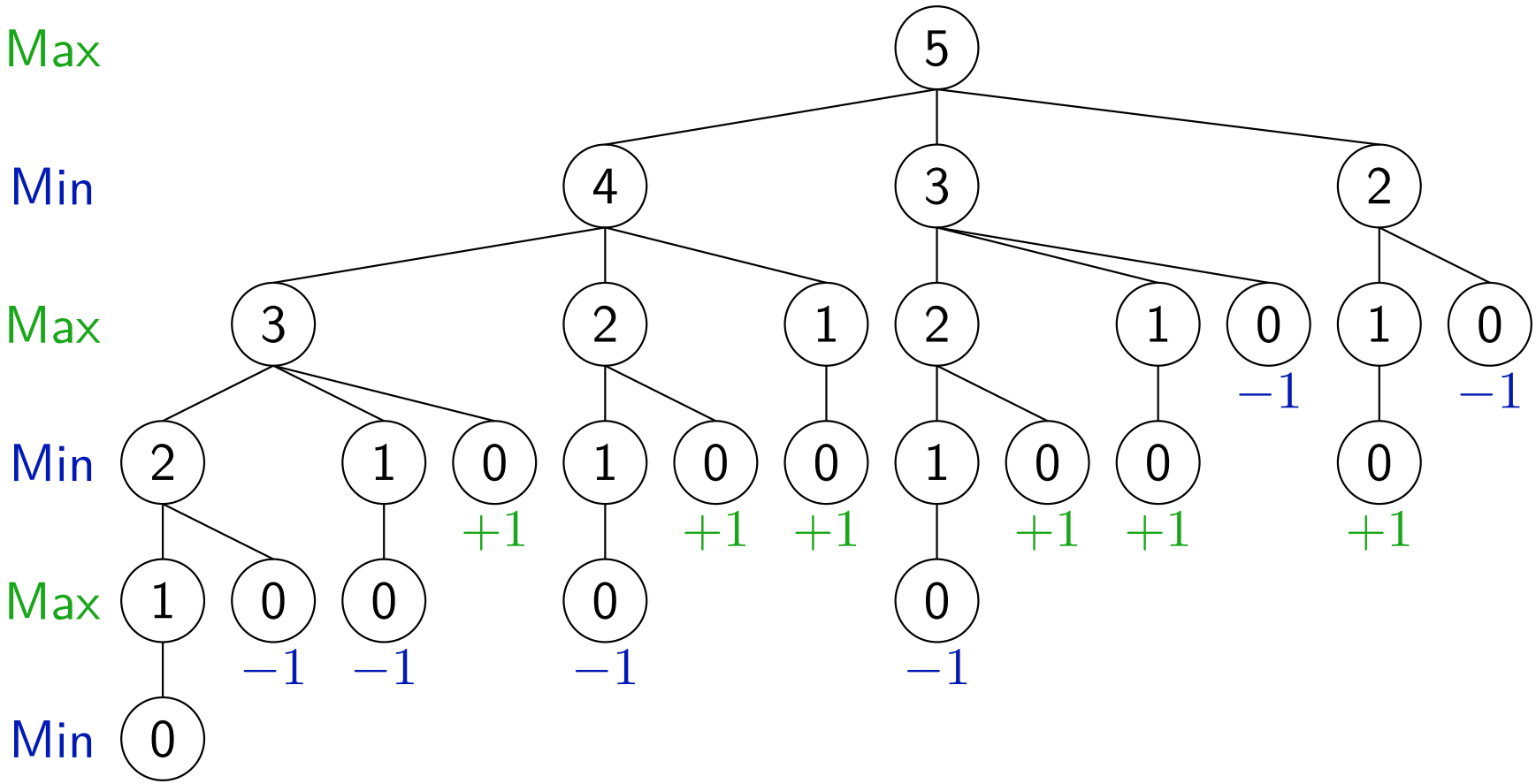
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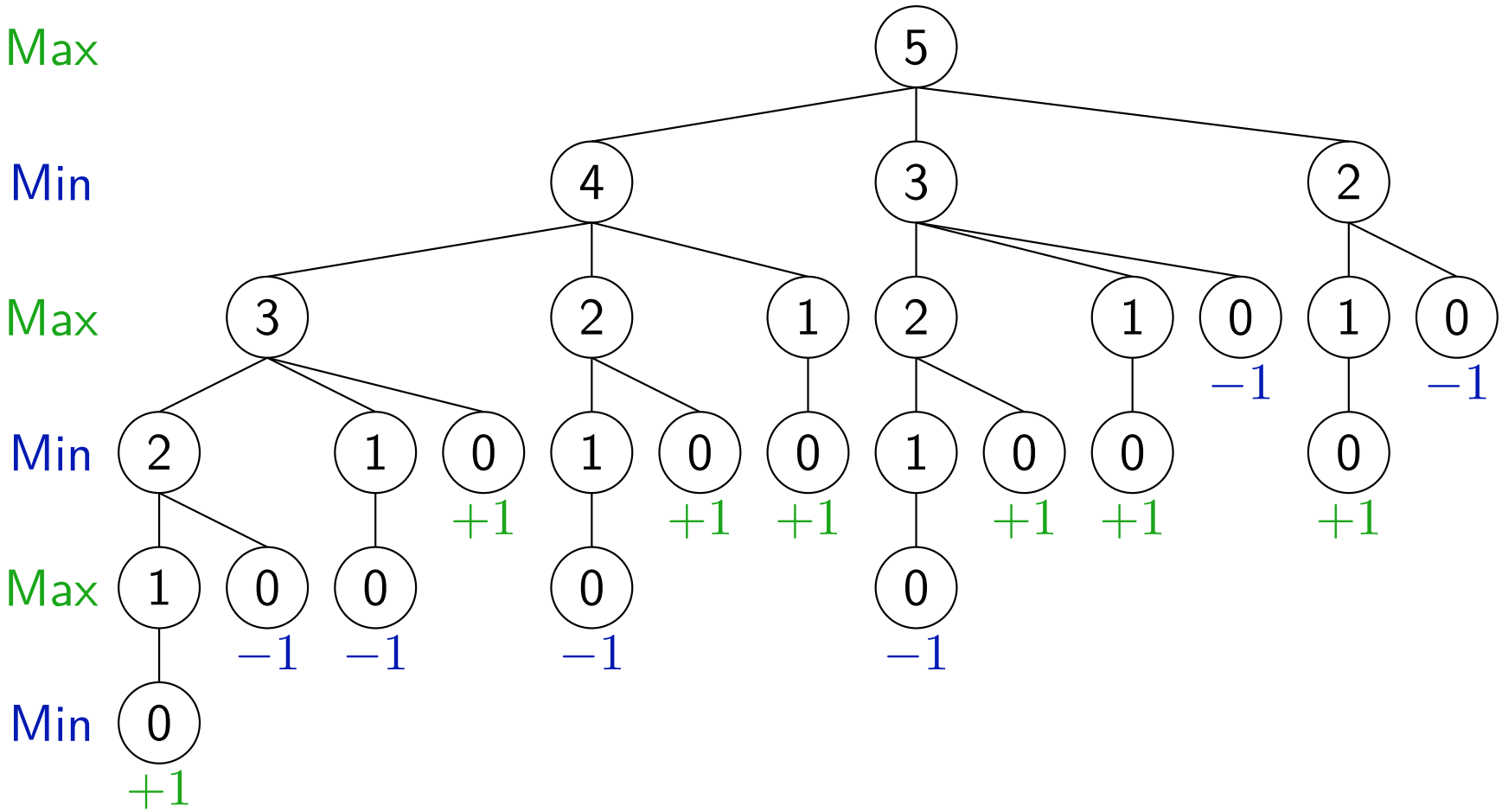
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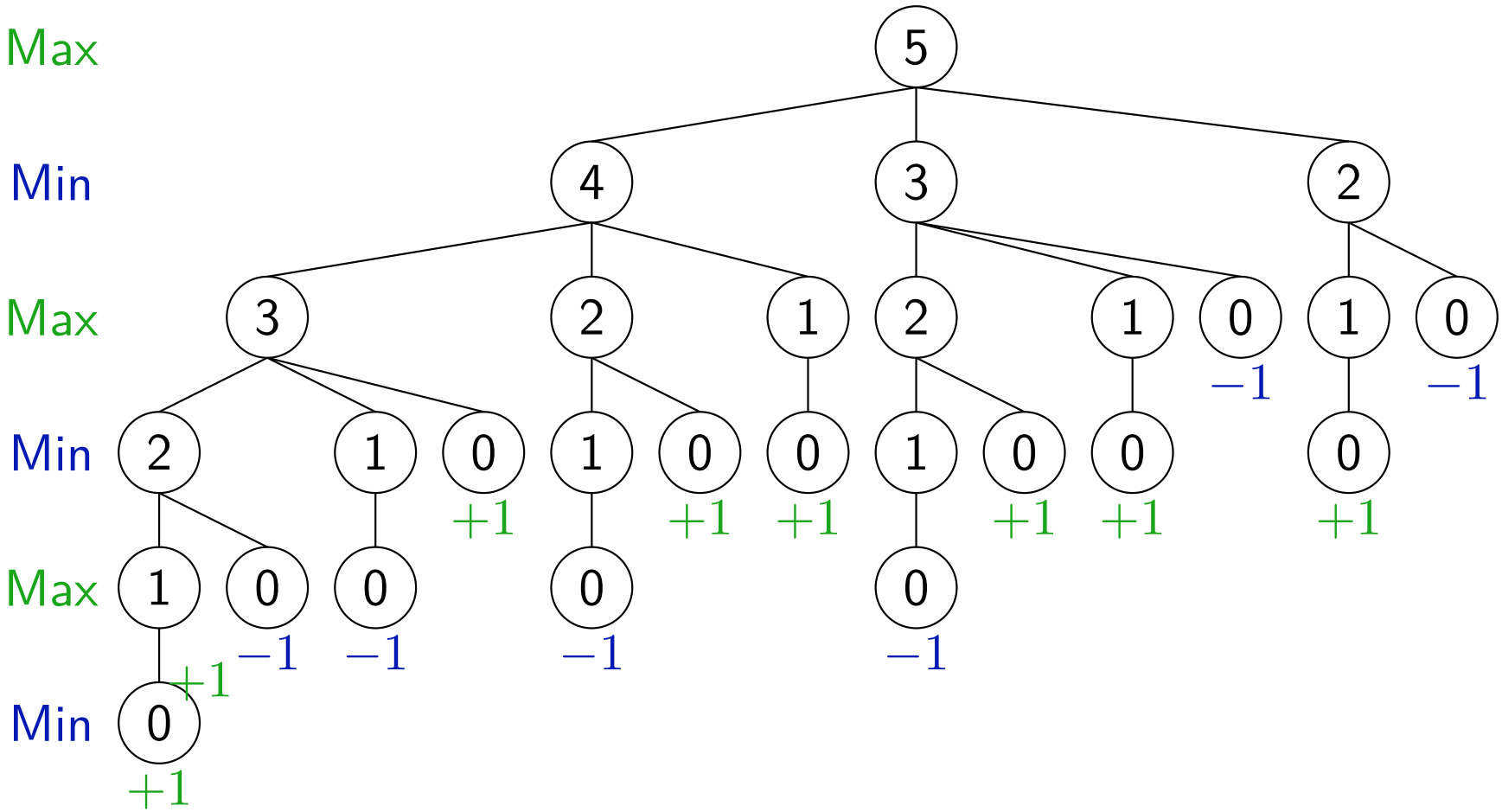
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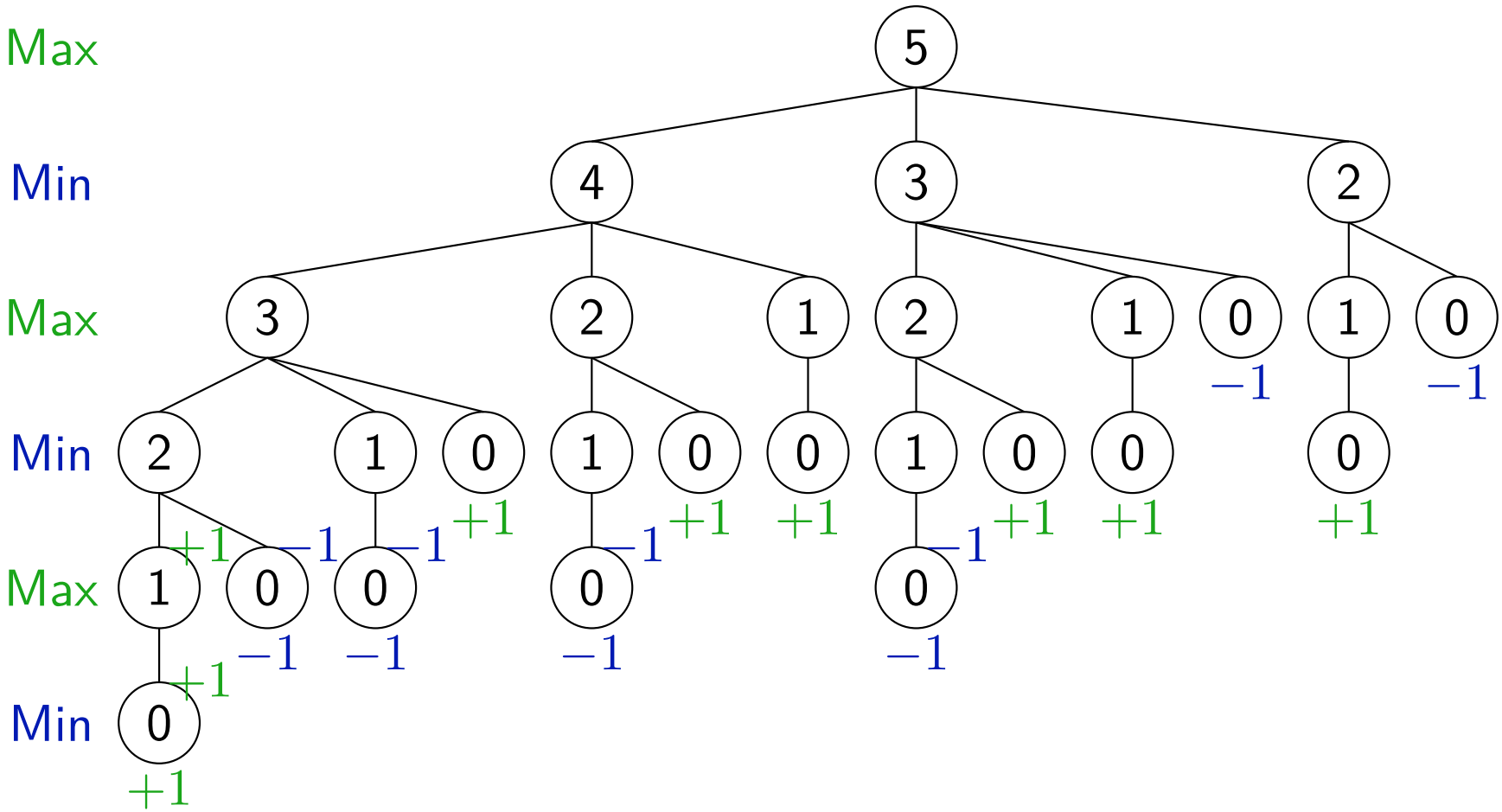
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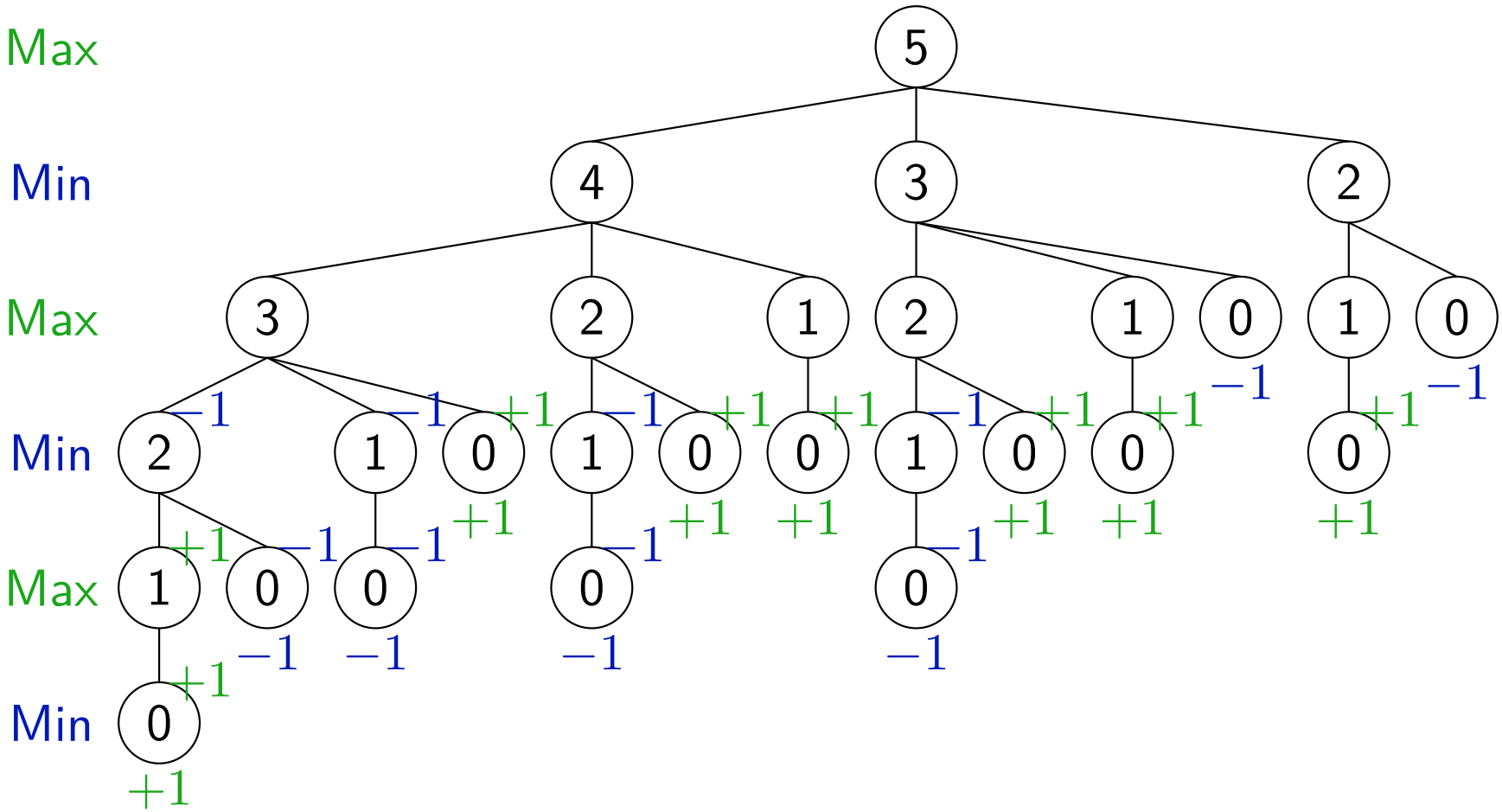
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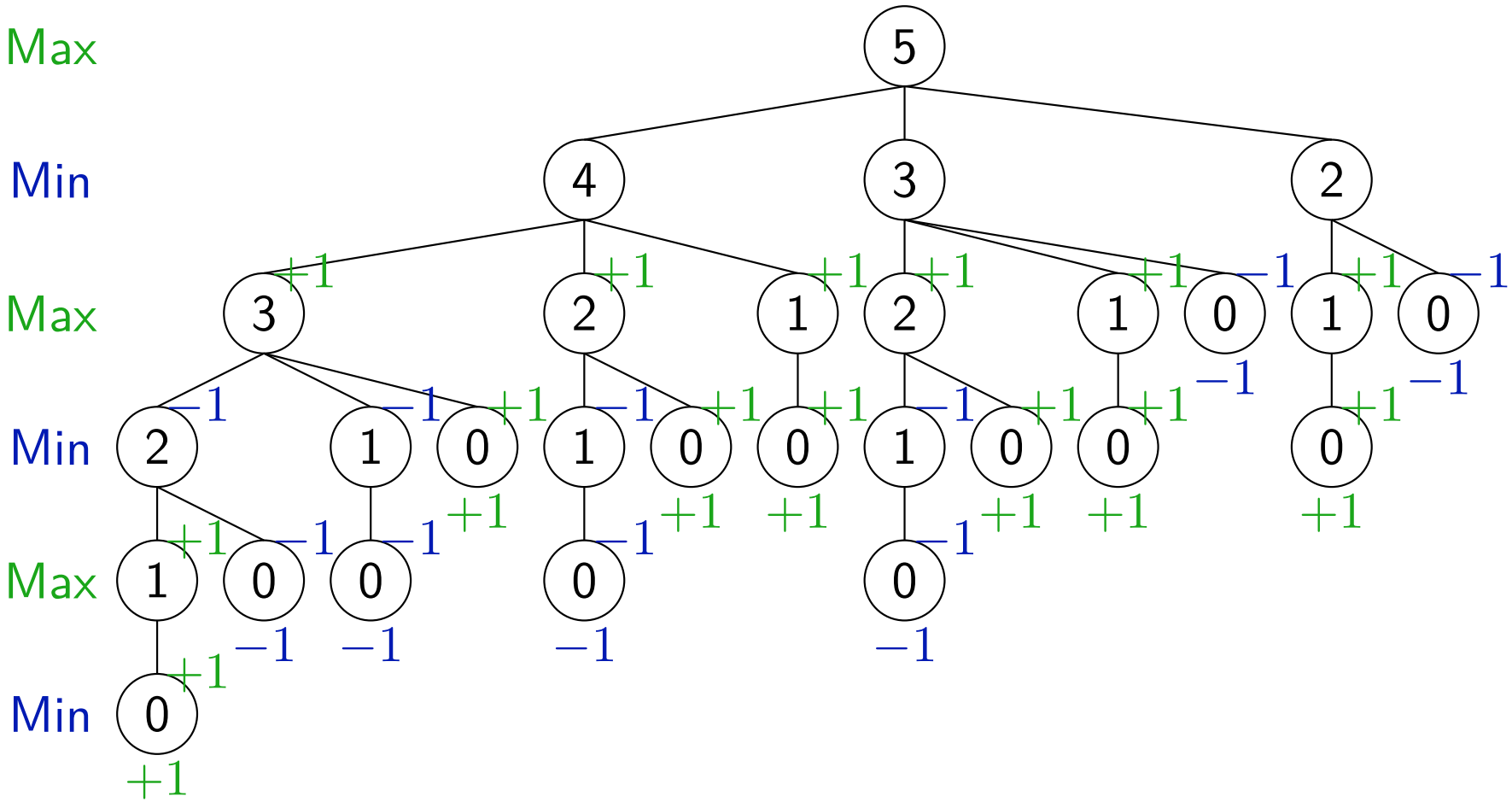
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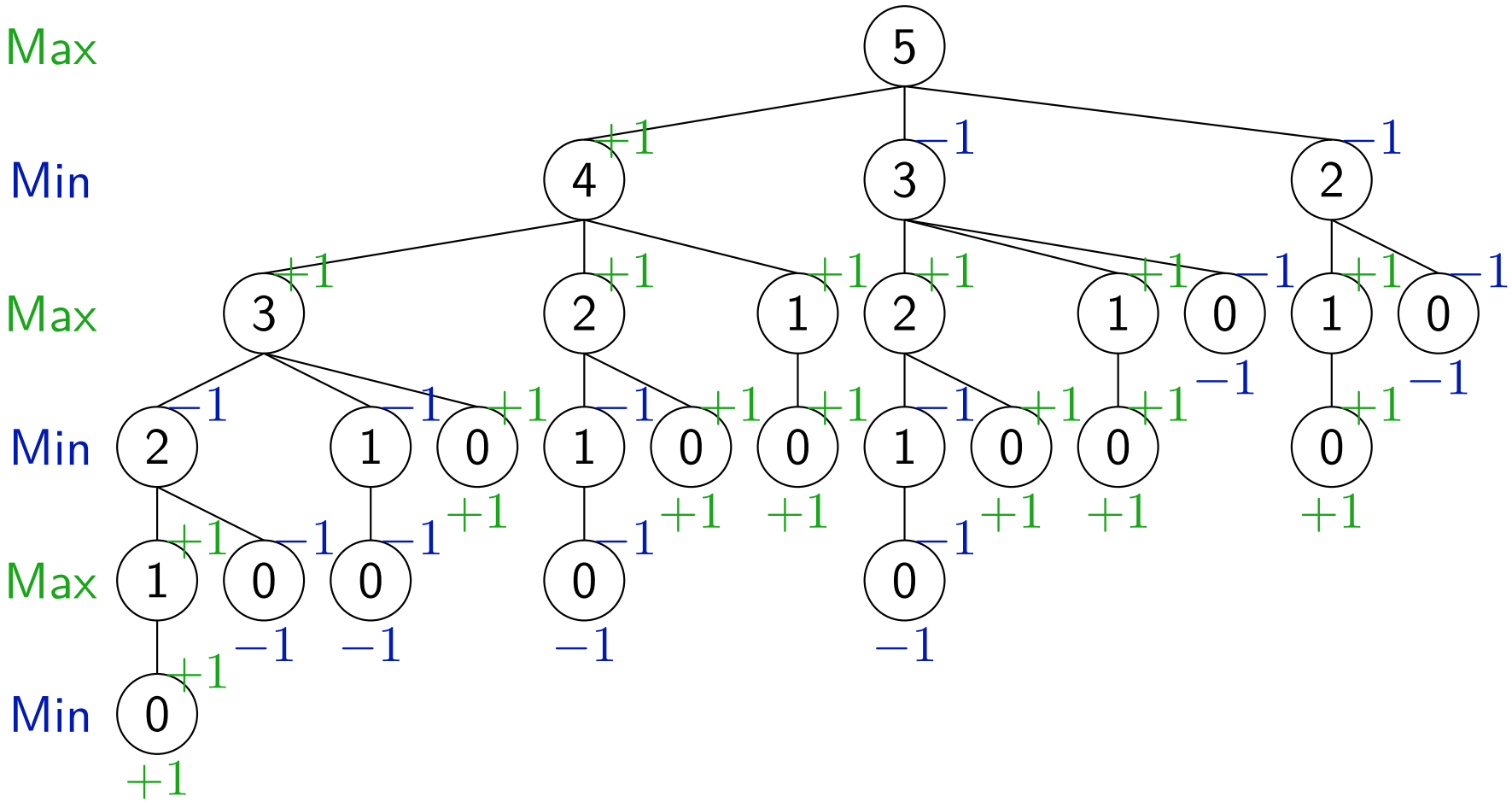
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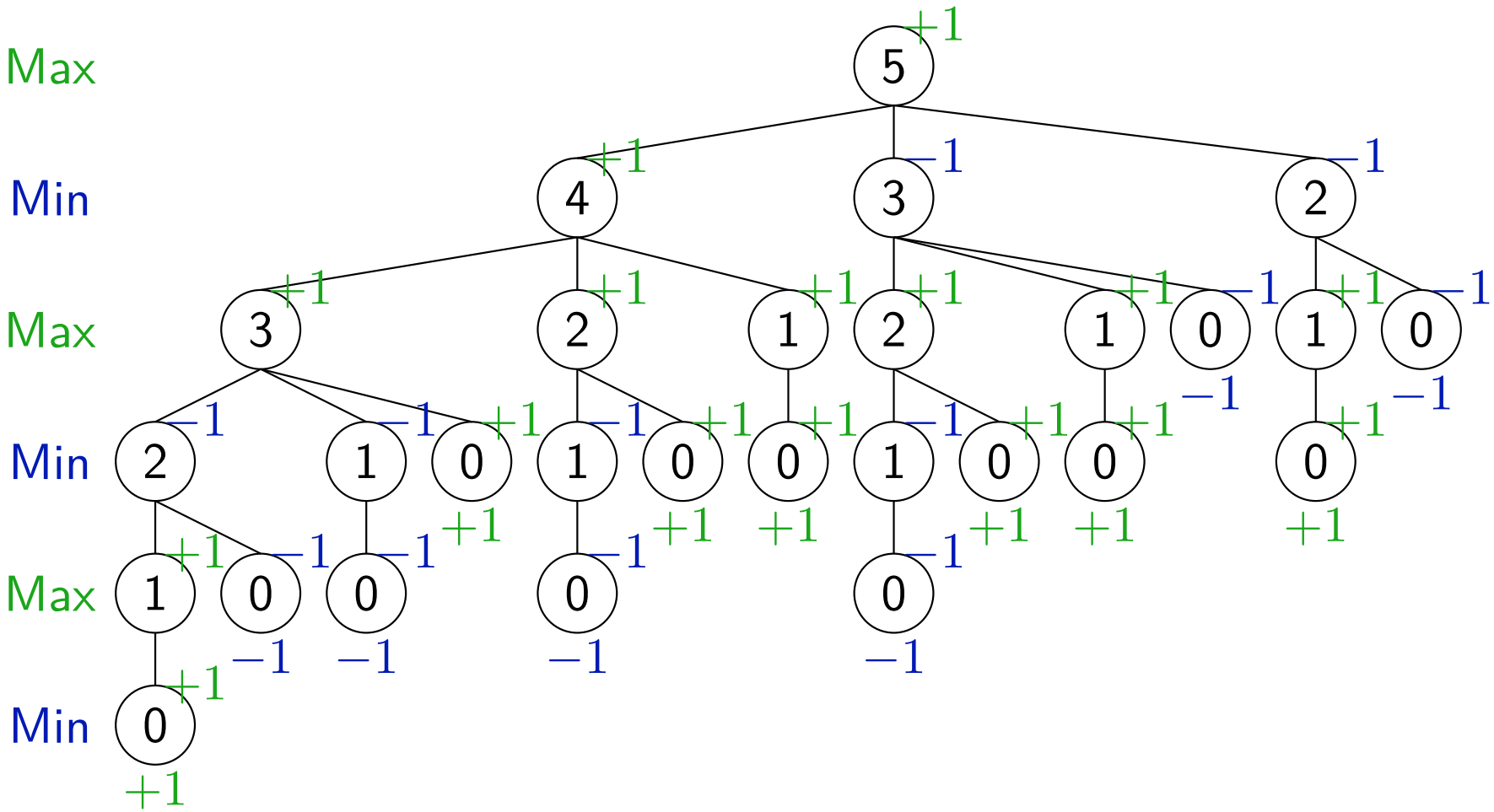

Minimax Algorithm

Minimax Algorithm

Computing Values

function max-value(s) returns utility value

1. if terminal-test(s) then return utility(s)
2. $v = -\infty$
3. for each a in actions(s) do
4. $\quad v = \max\{v, \text{min-value}(\text{result}(s, a))\}$
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Decision for Max

return $\underset{a \in \text{actions}(s)}{\arg\max}$ min-value(result(s, a))

About Minimax Search

About Minimax Search

Observations

- minimax algorithm guarantees optimal choice of Max if Min plays optimally

About Minimax Search

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  let player i select node that maximises i-th component

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- in games which are not zero-sum other strategies may be optimal even for only two players

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Goal improve run time
without sacrificing optimality

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update α and β whenever possible
prune max-nodes $< \alpha$
prune min-nodes $> \beta$

Example Alpha-Beta Pruning

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Max

Min

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Min

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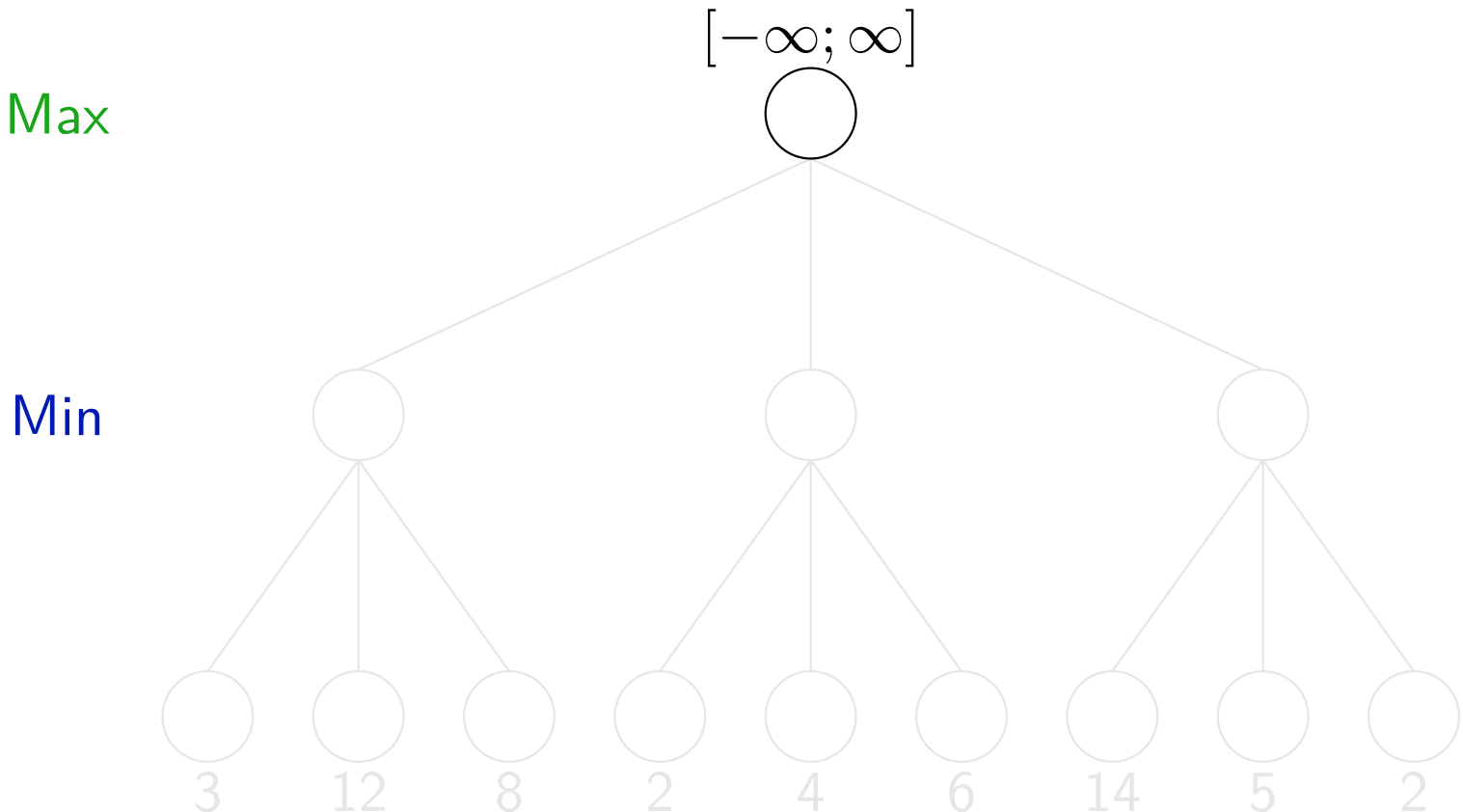

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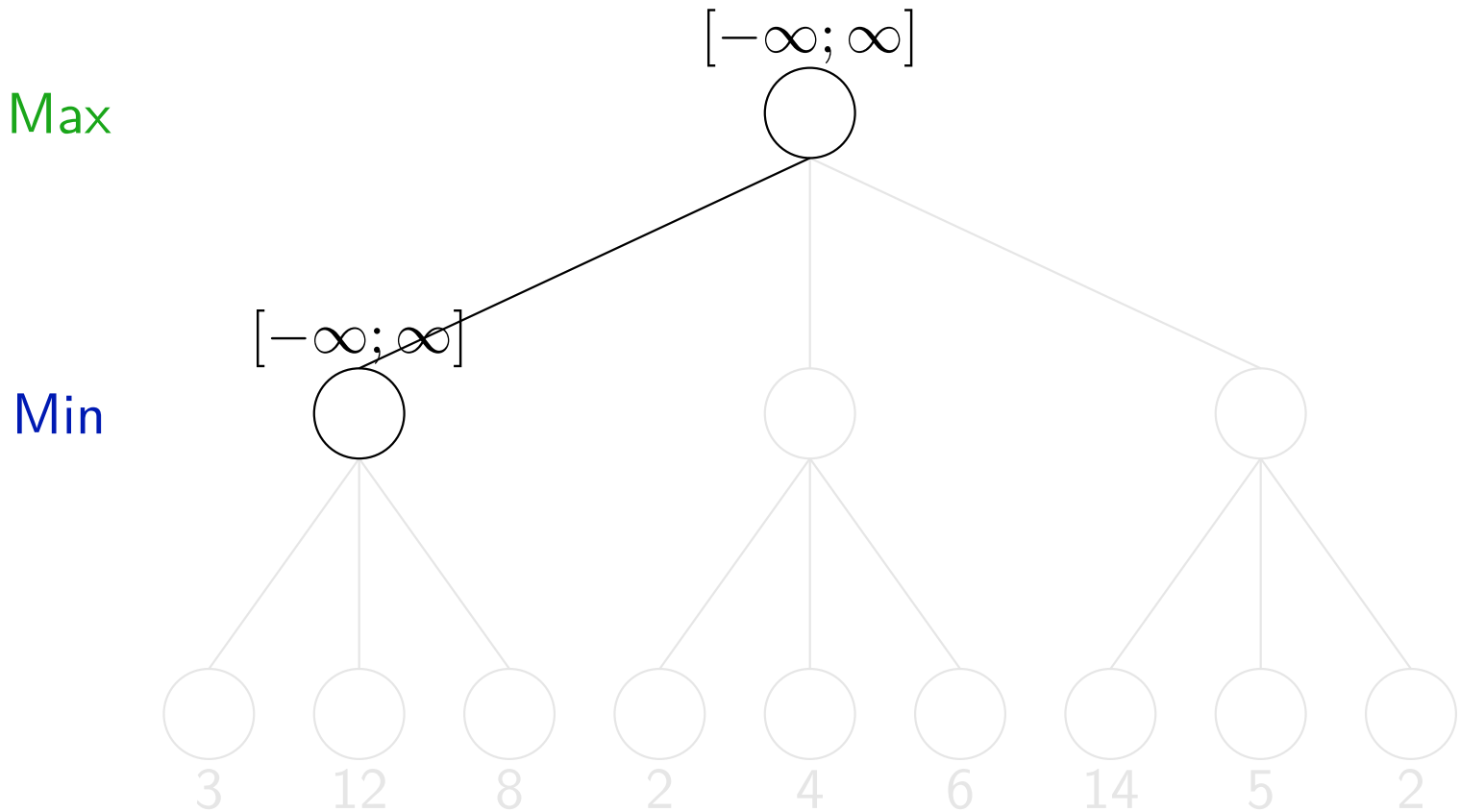

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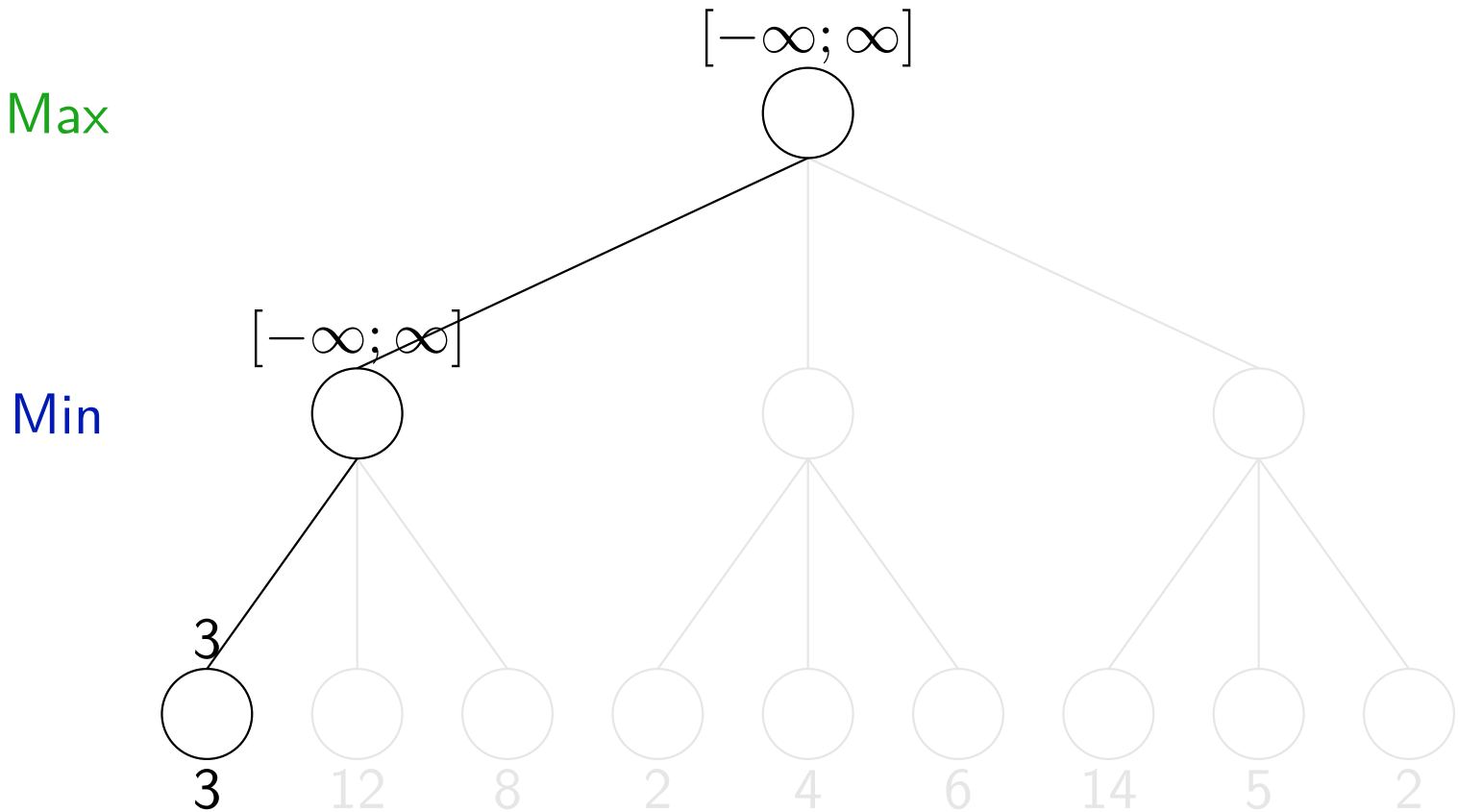

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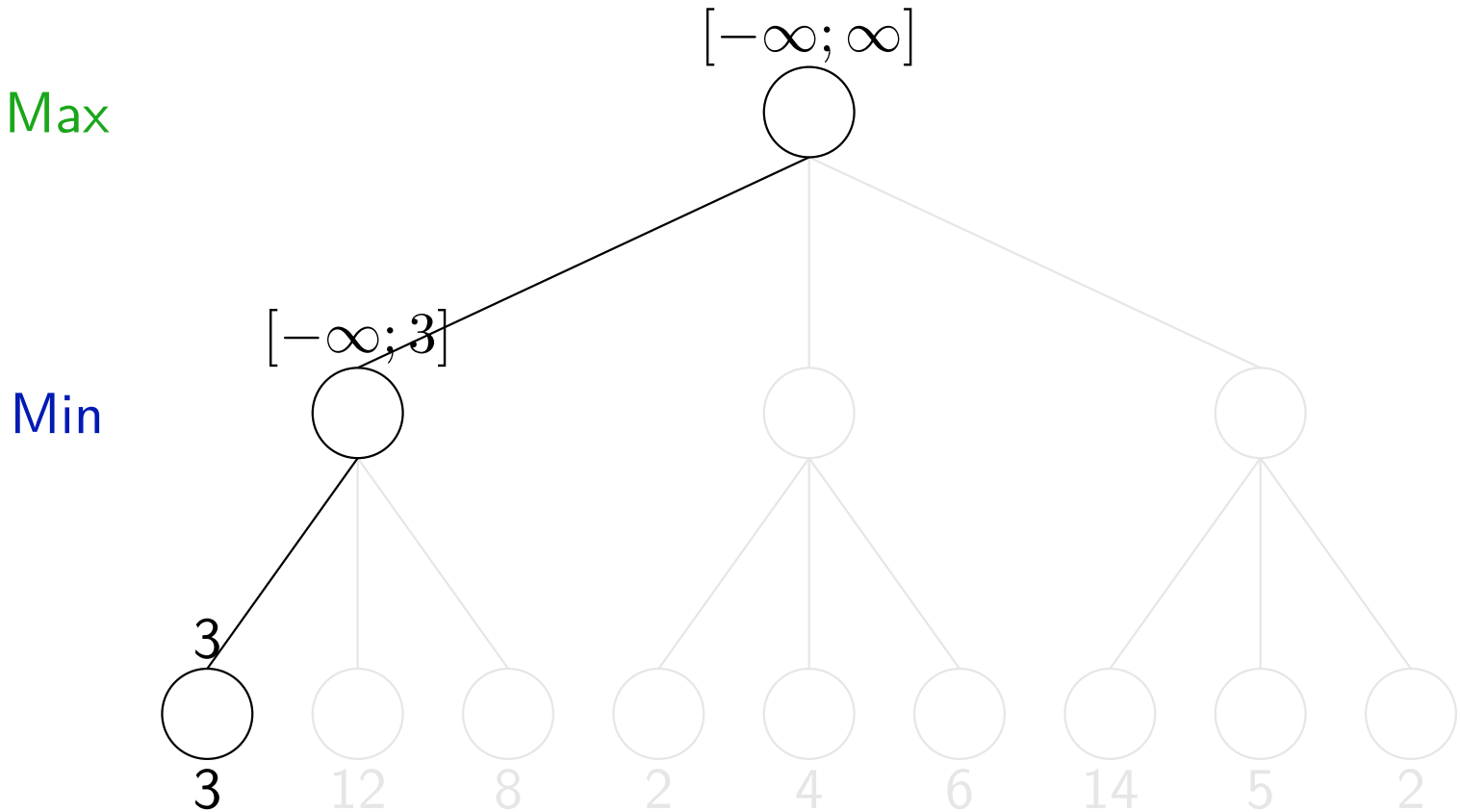

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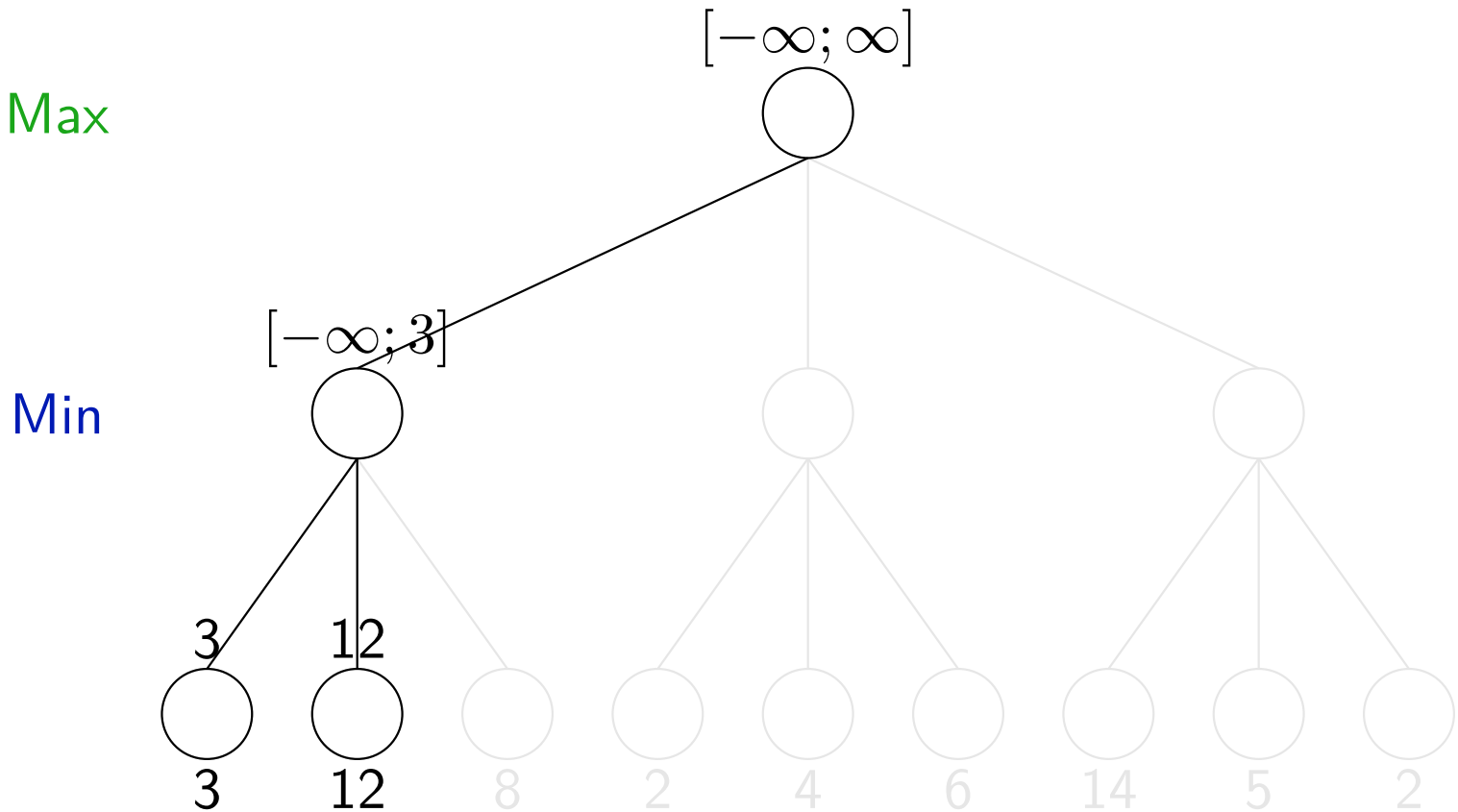

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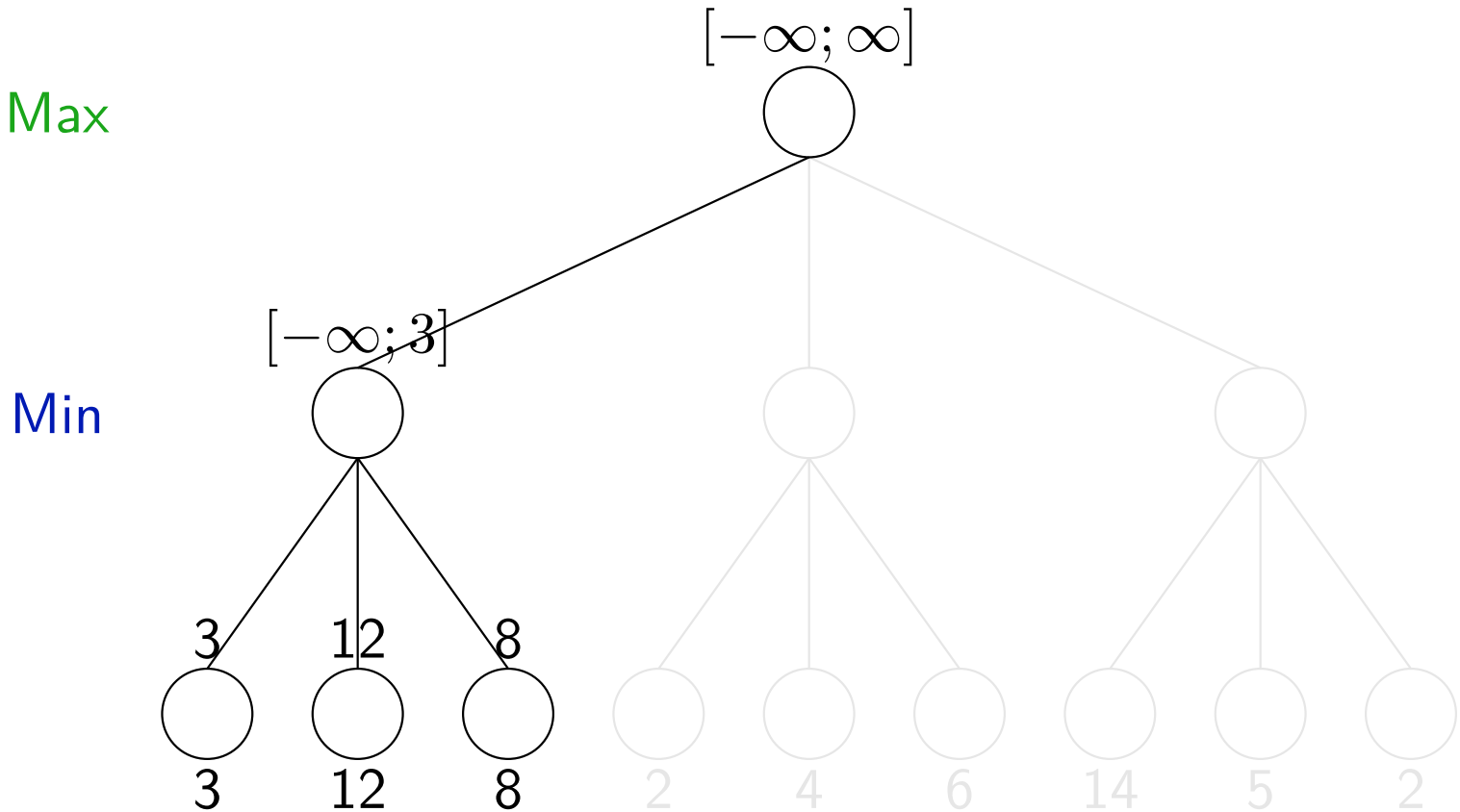

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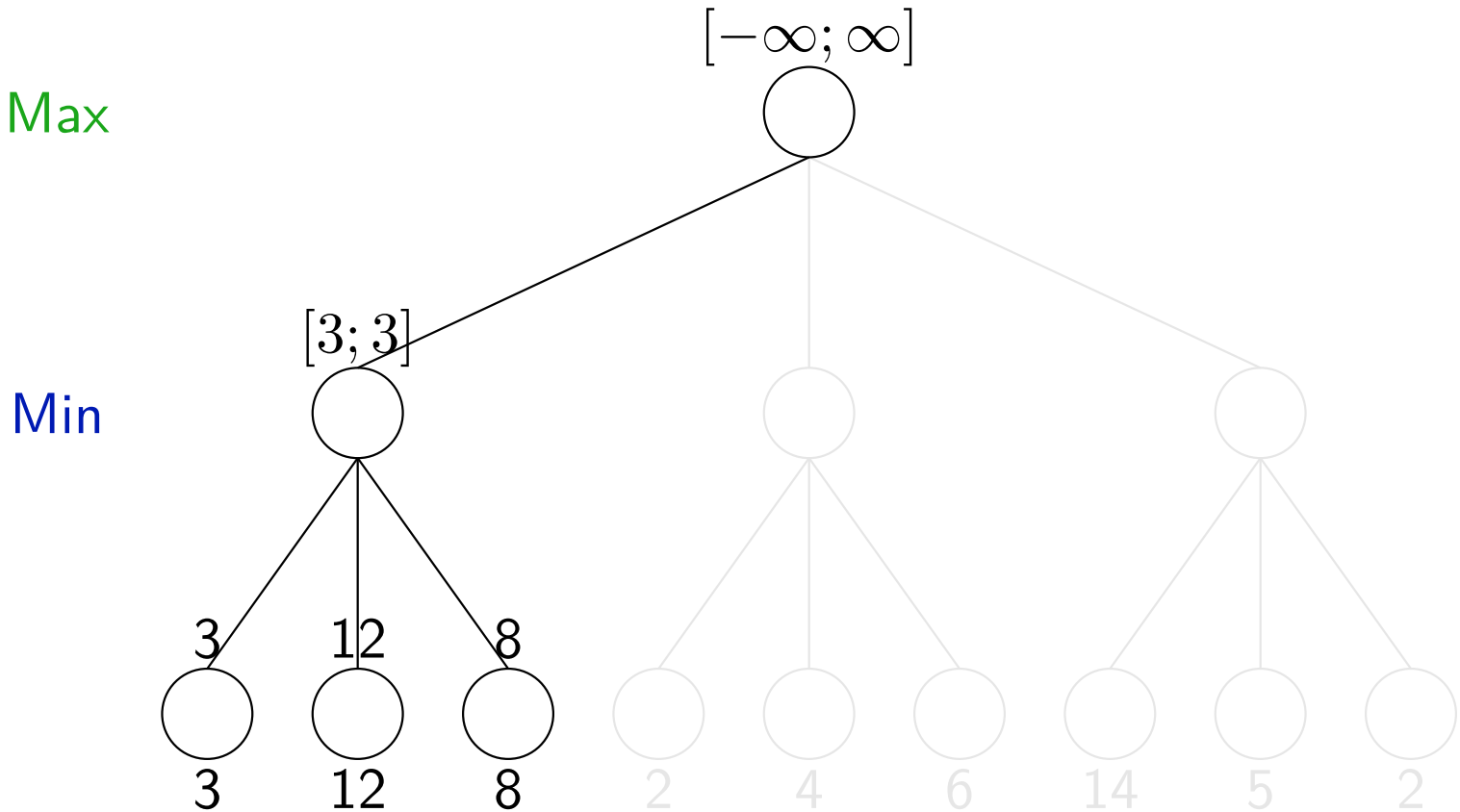

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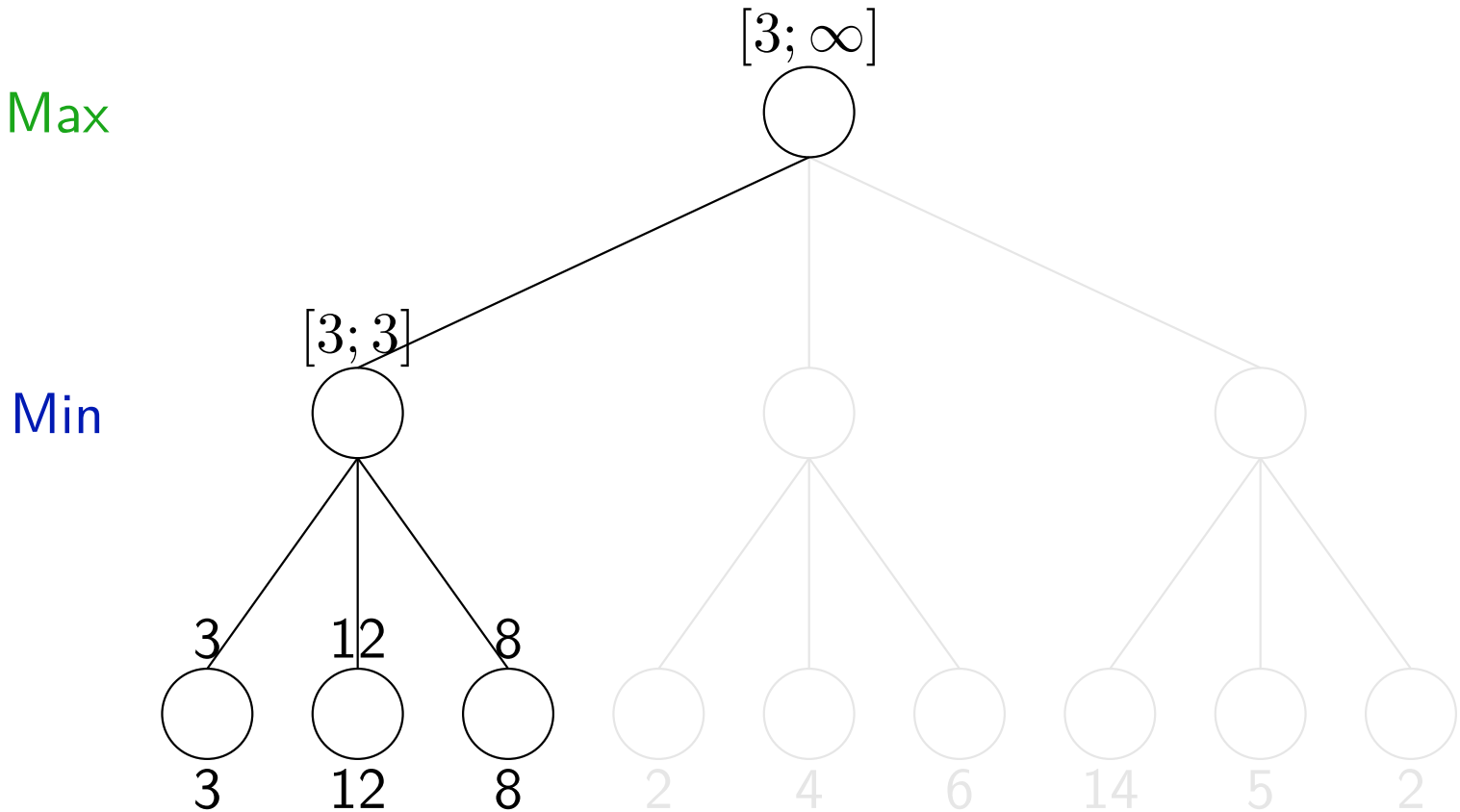

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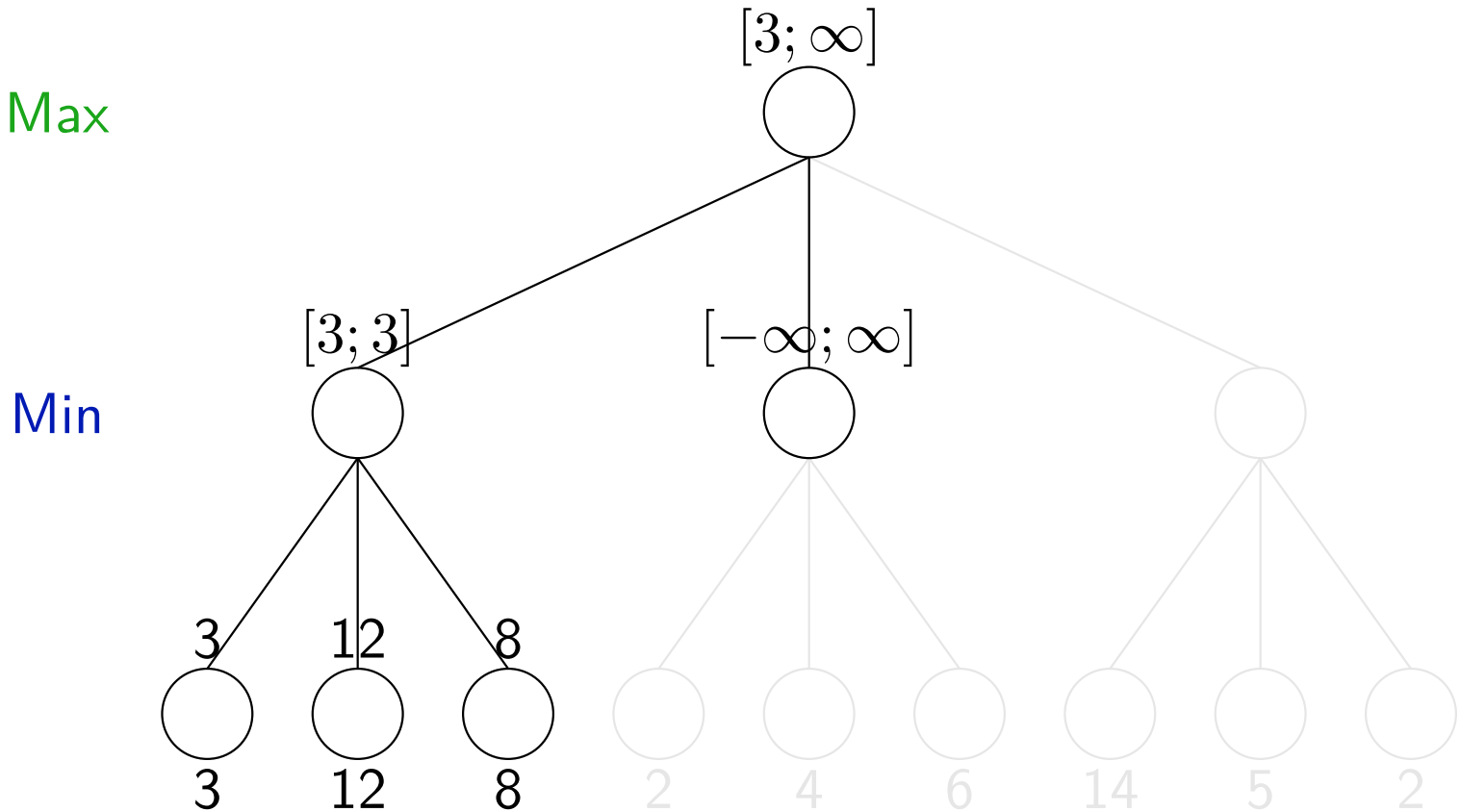

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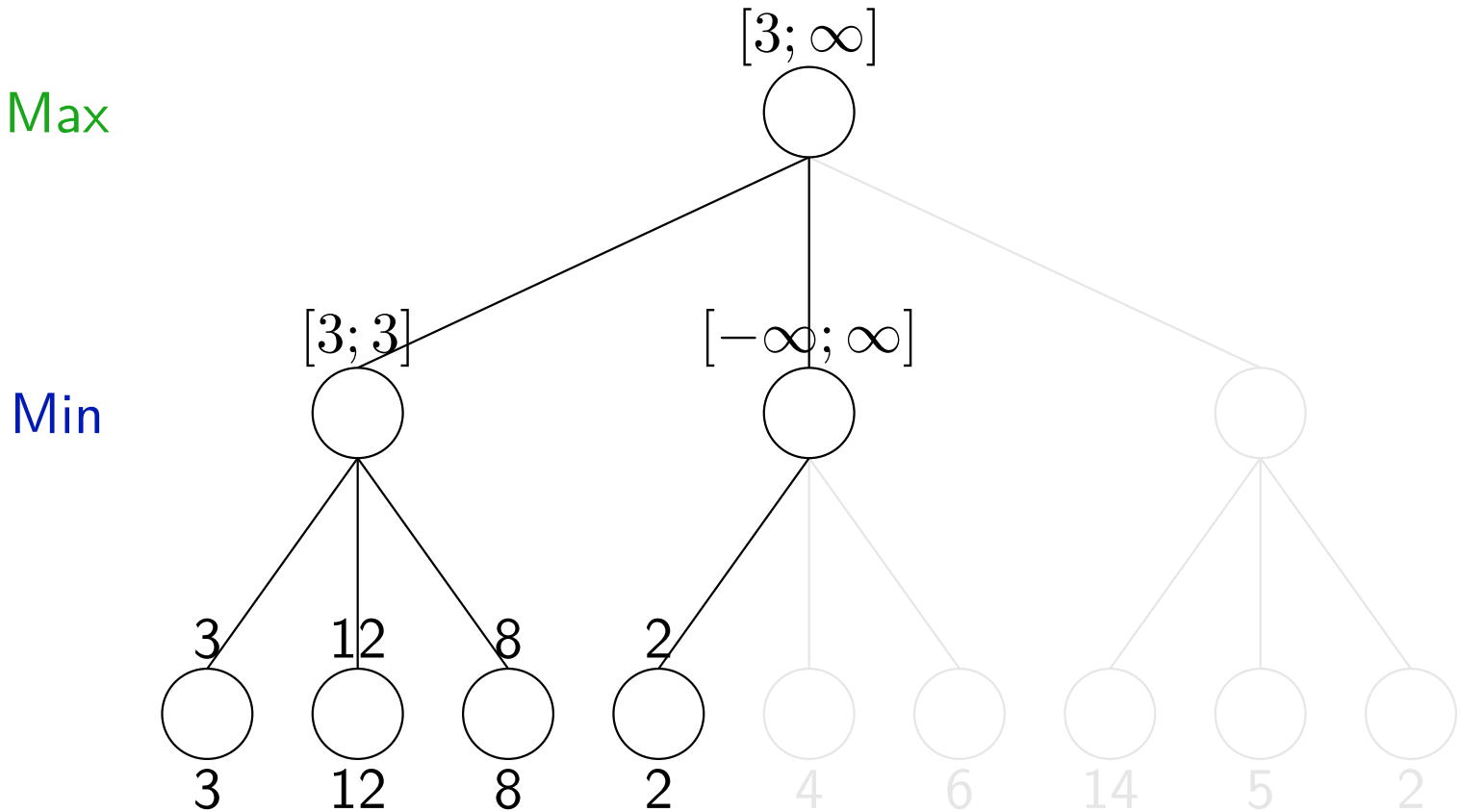

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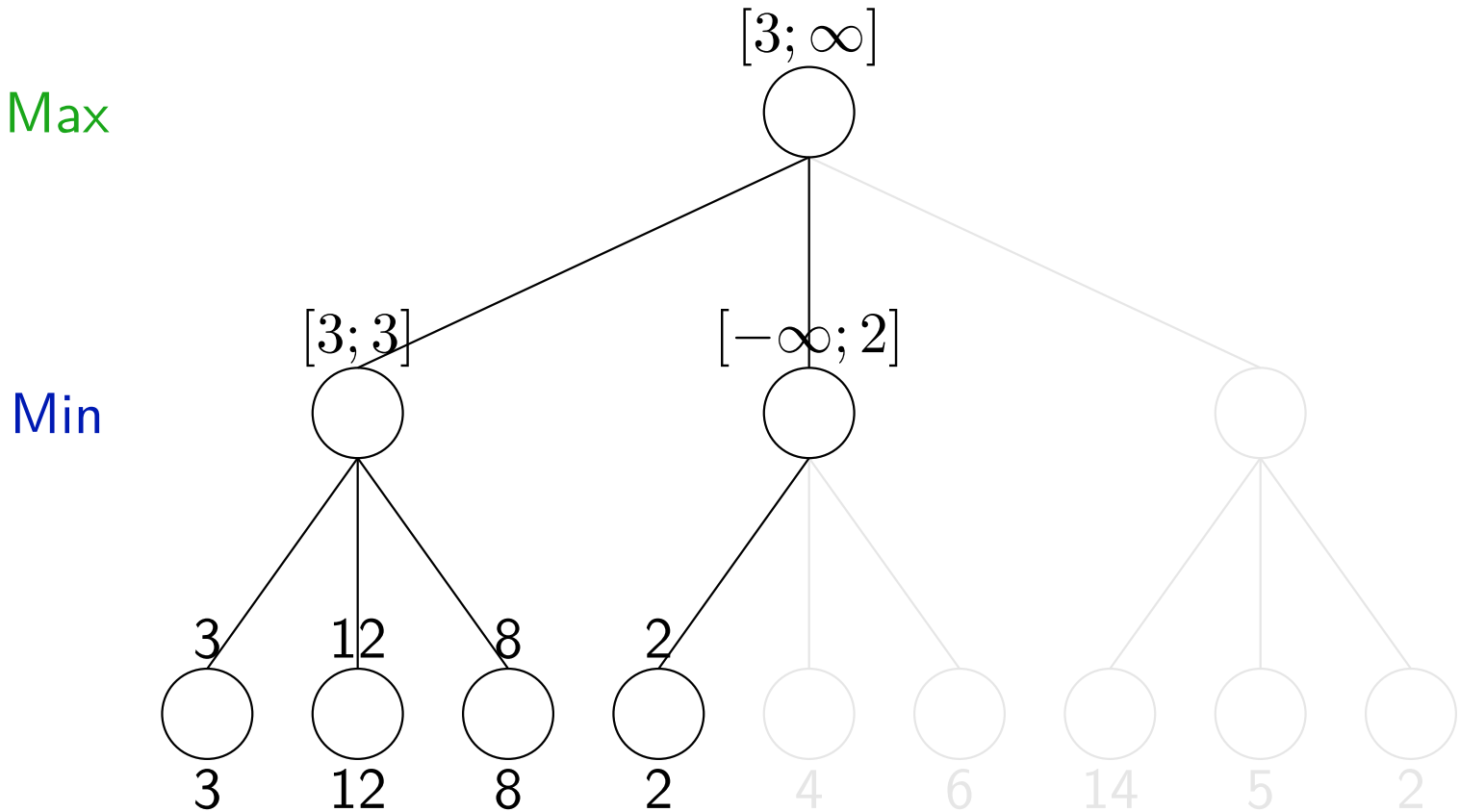

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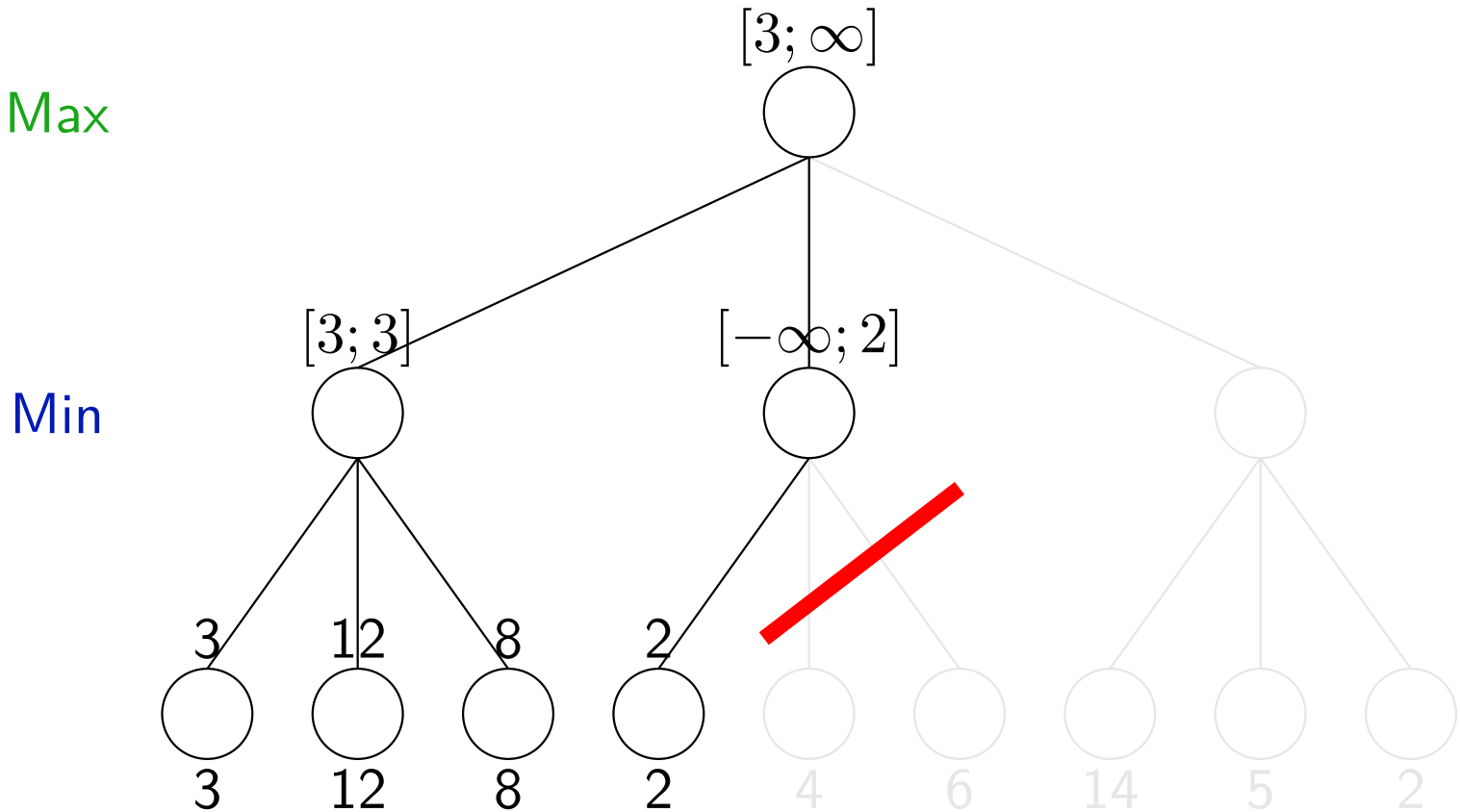

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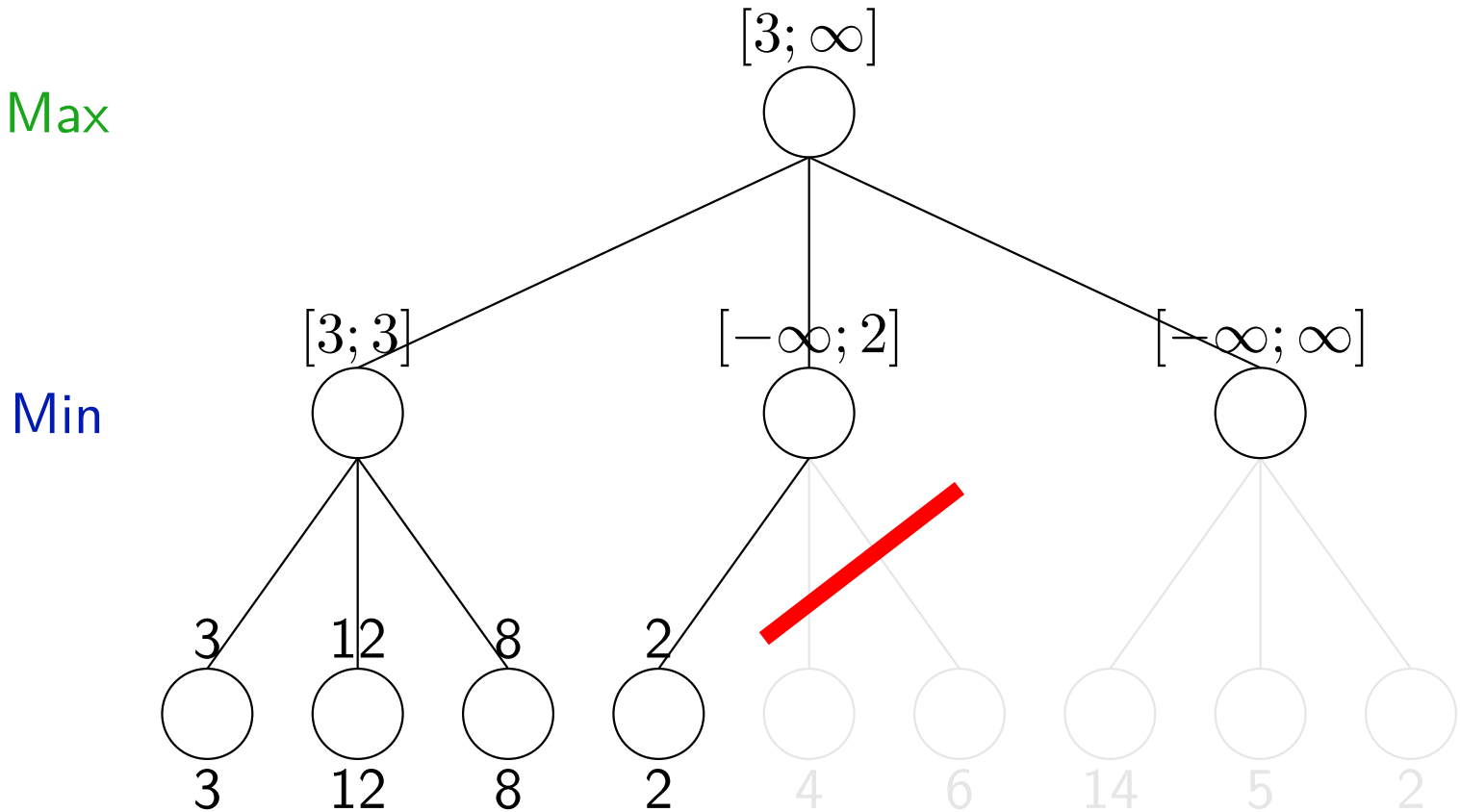

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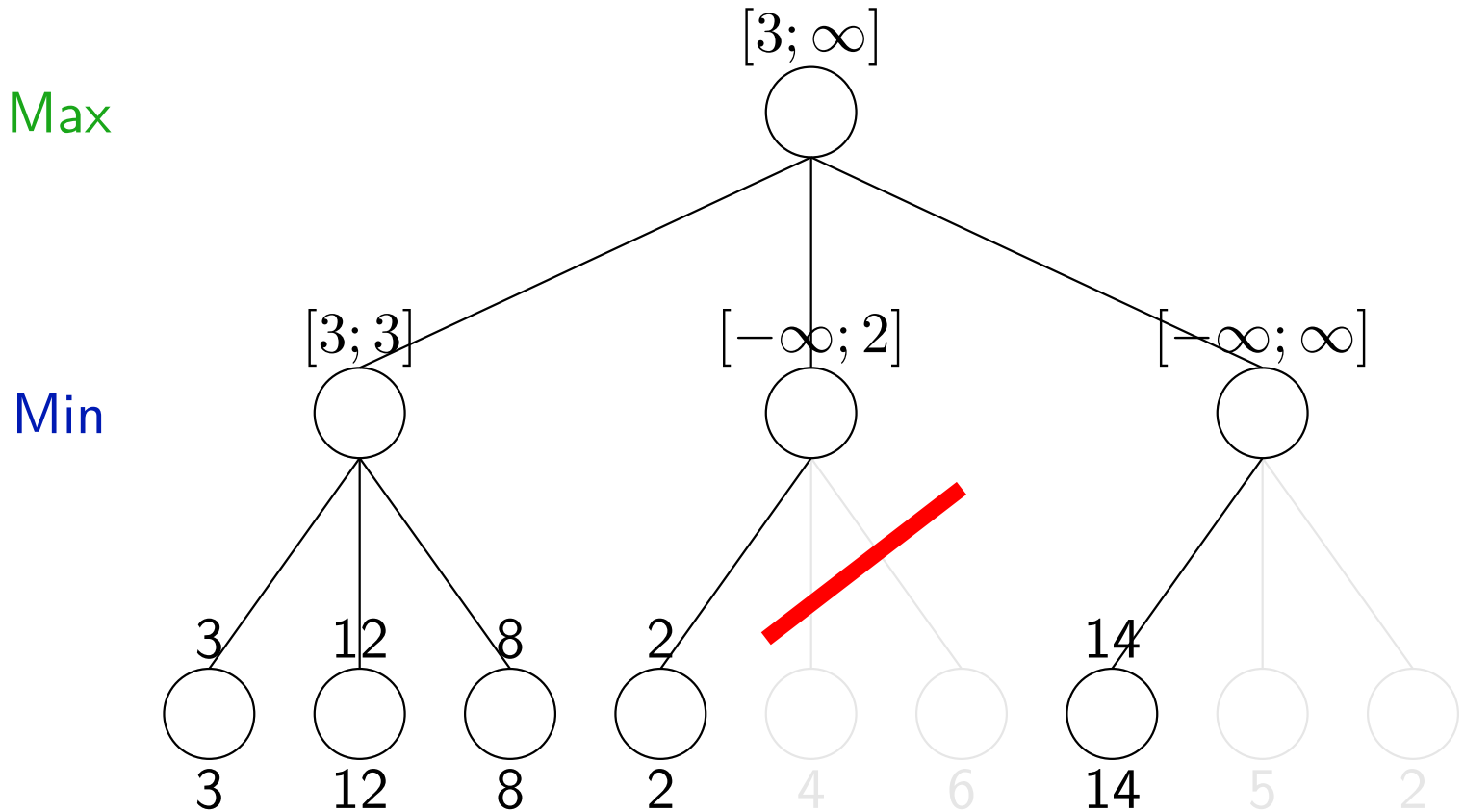

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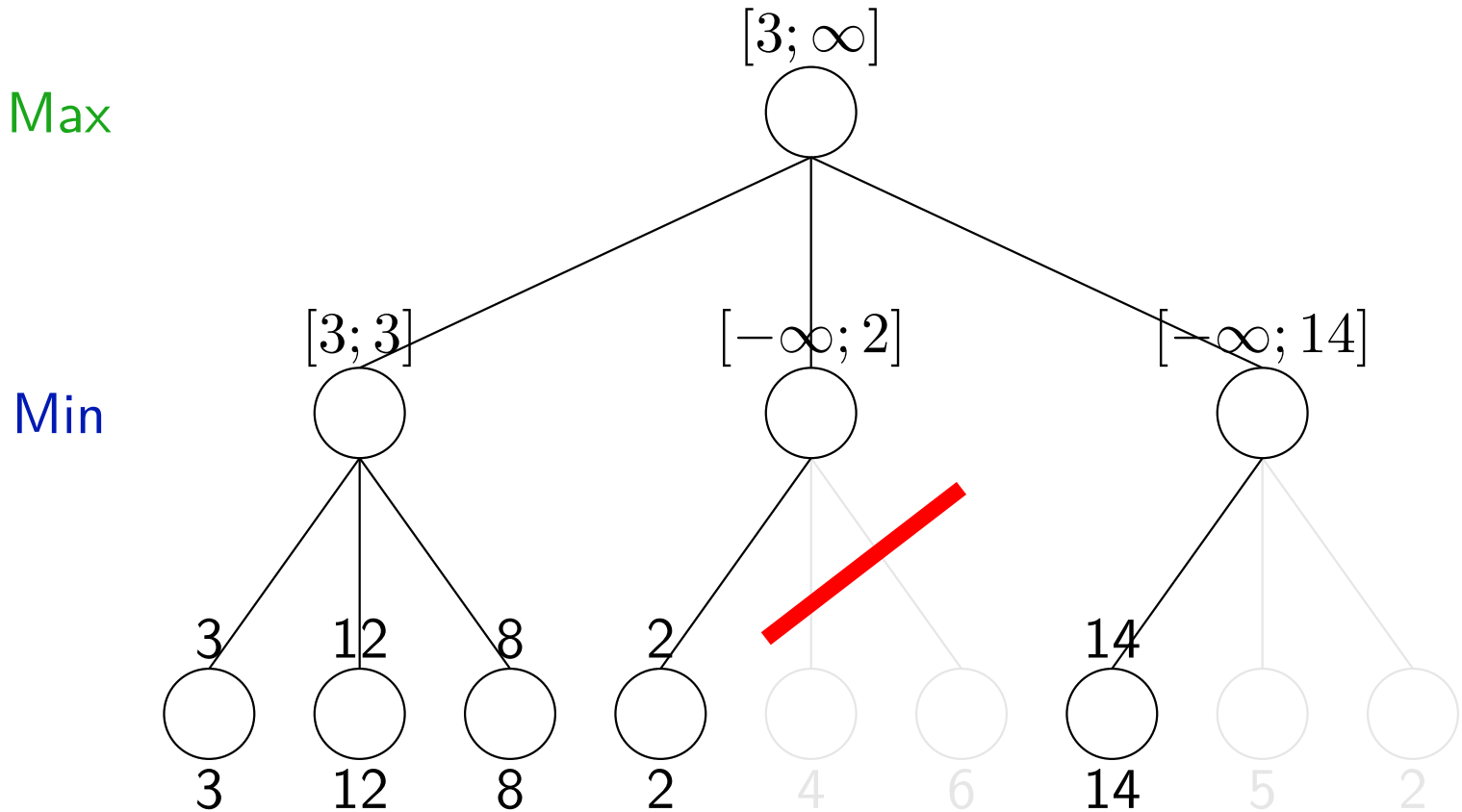

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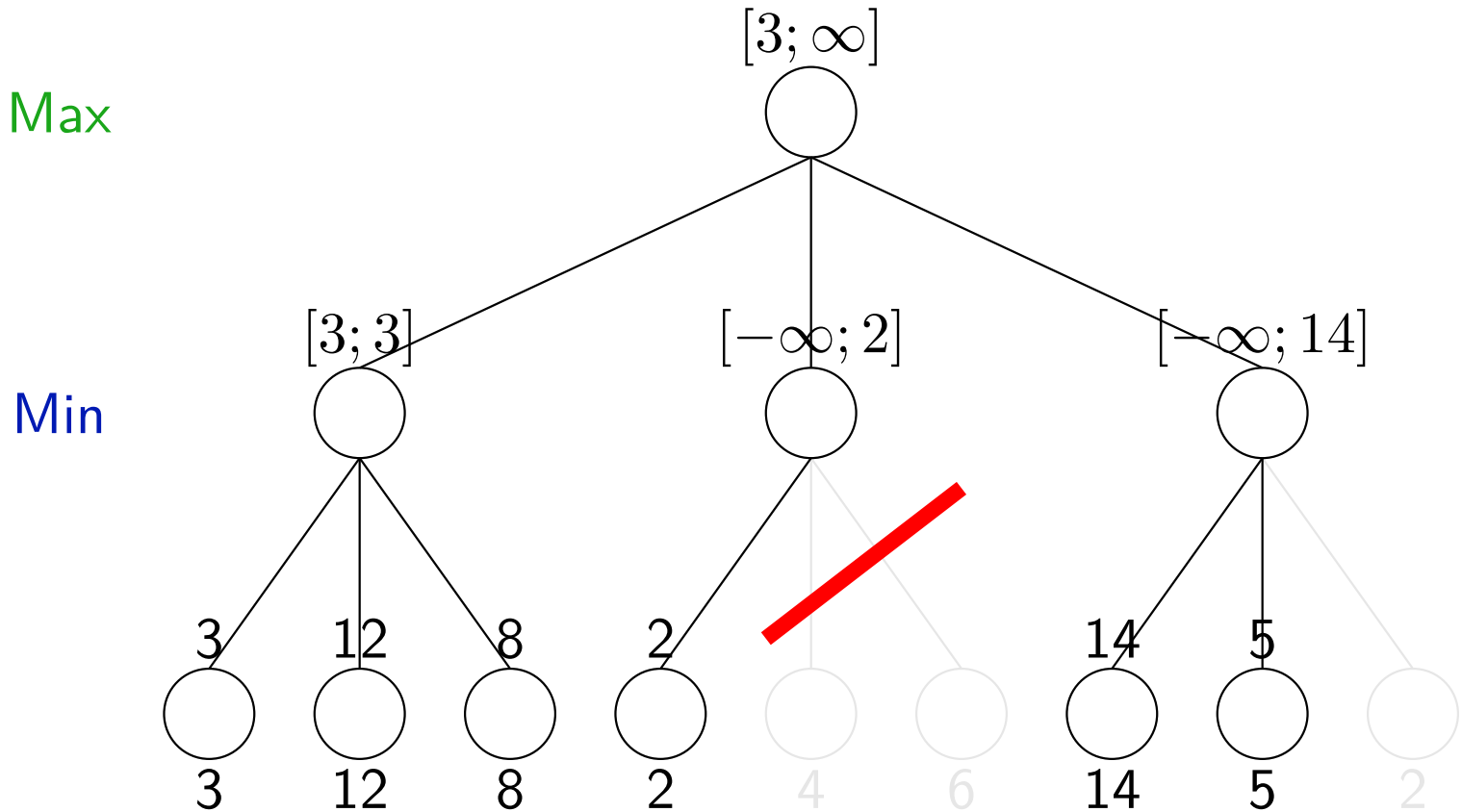

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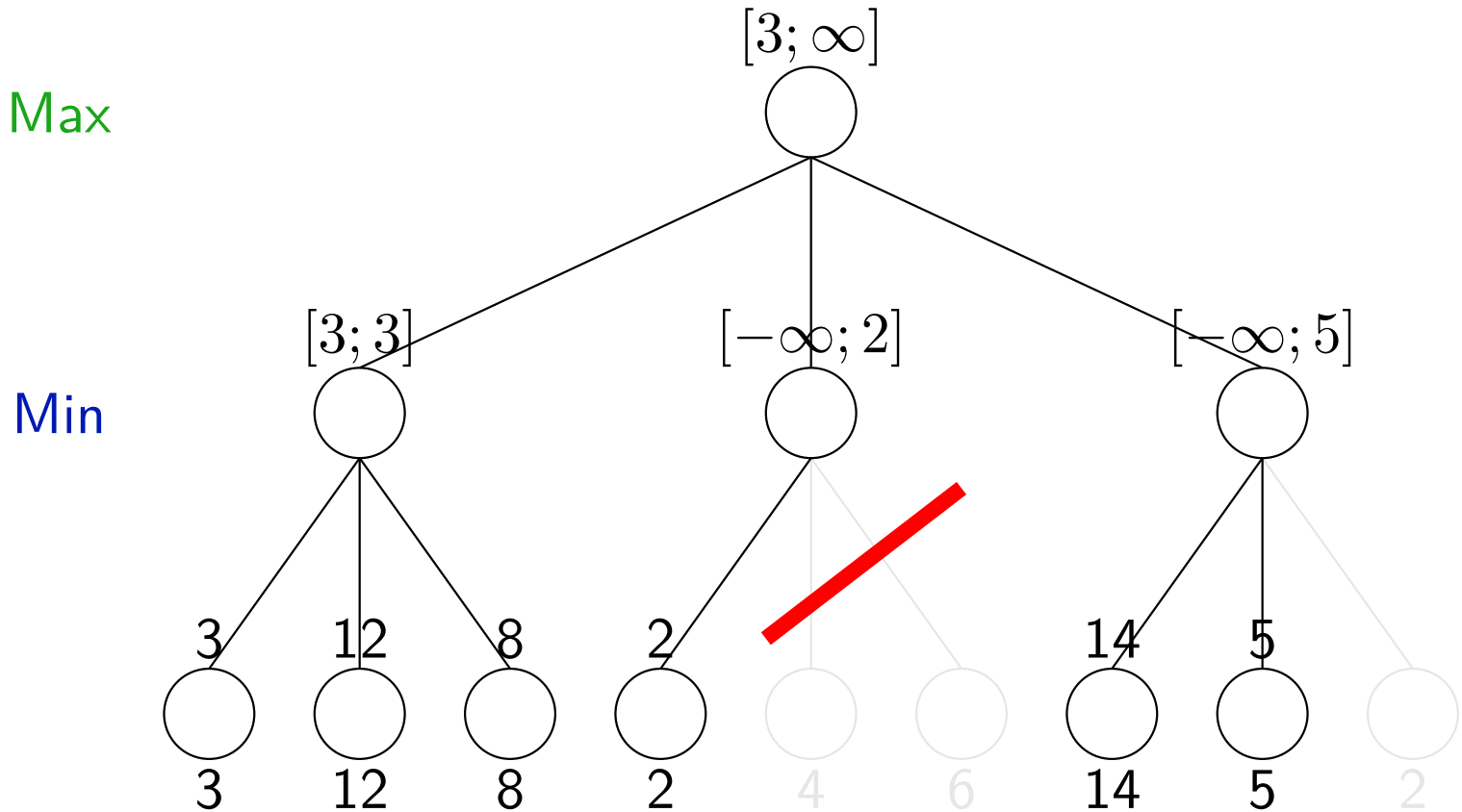

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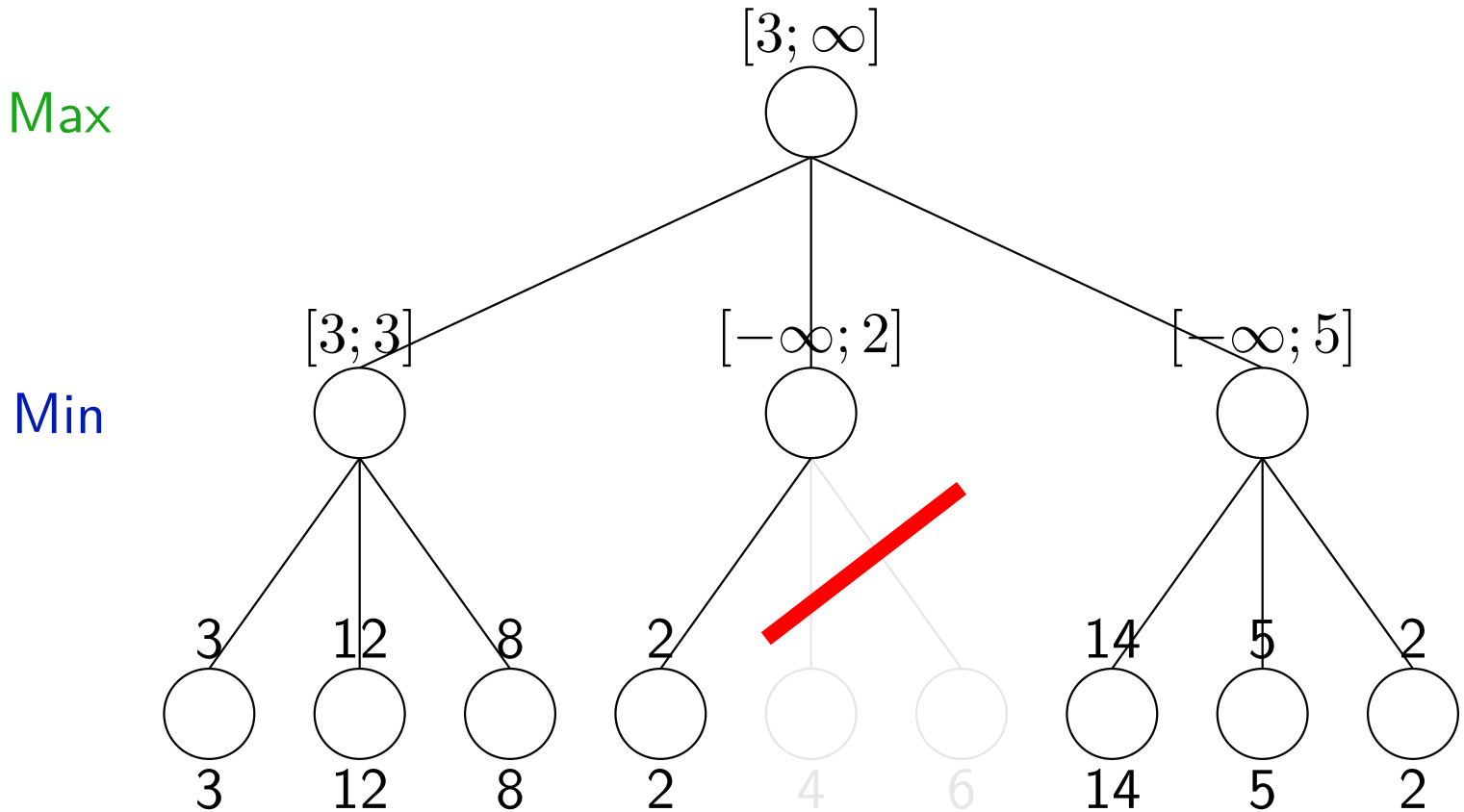

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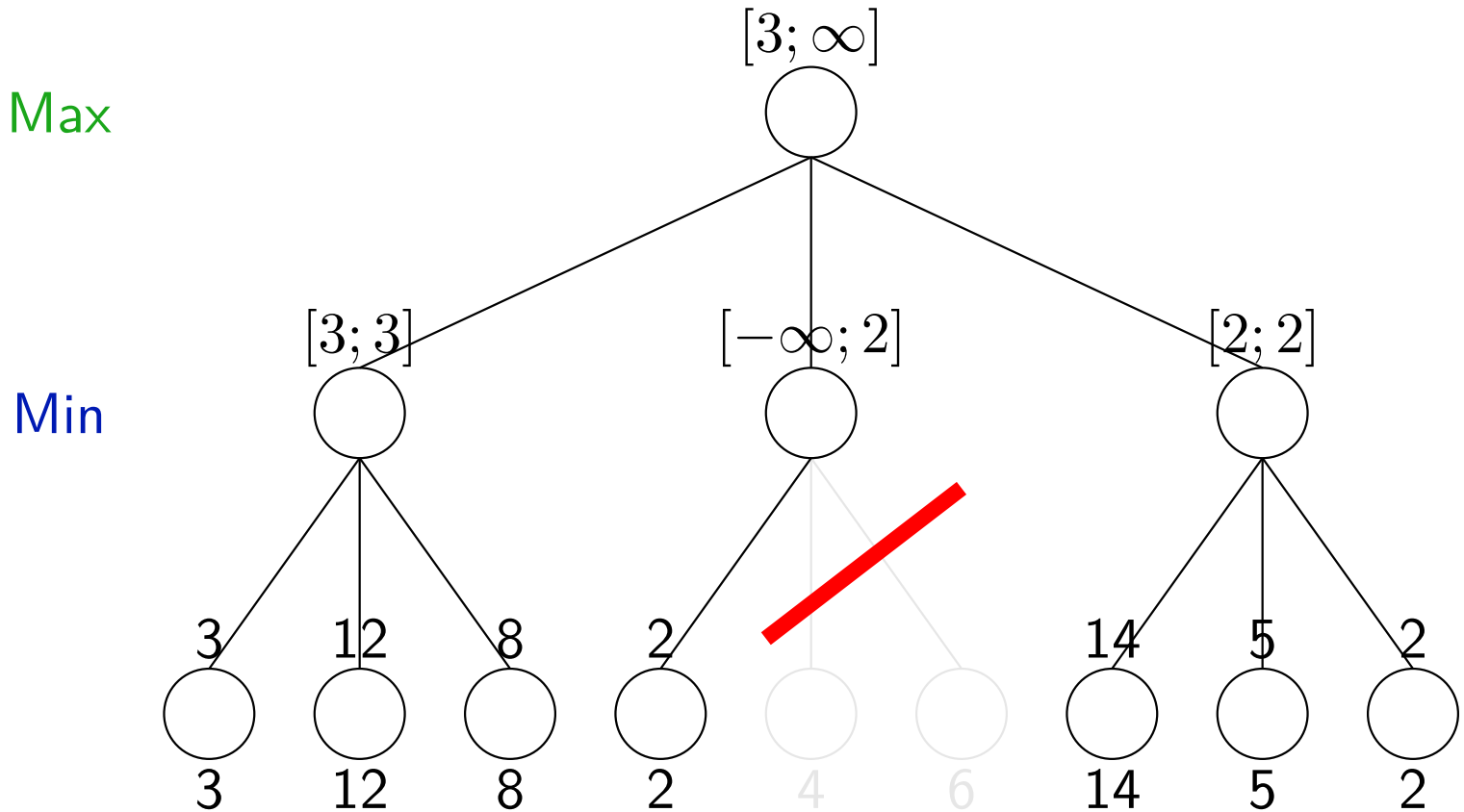

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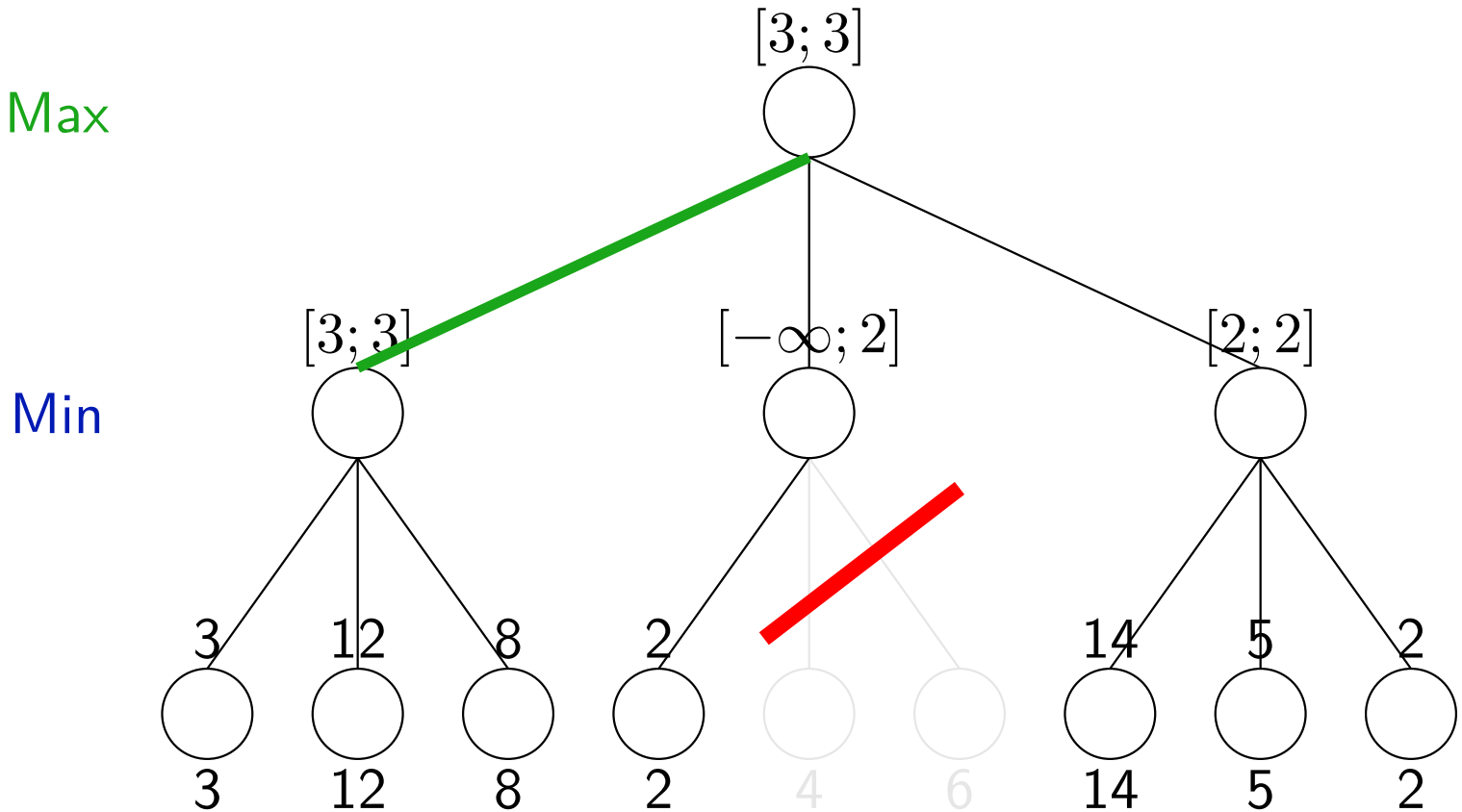

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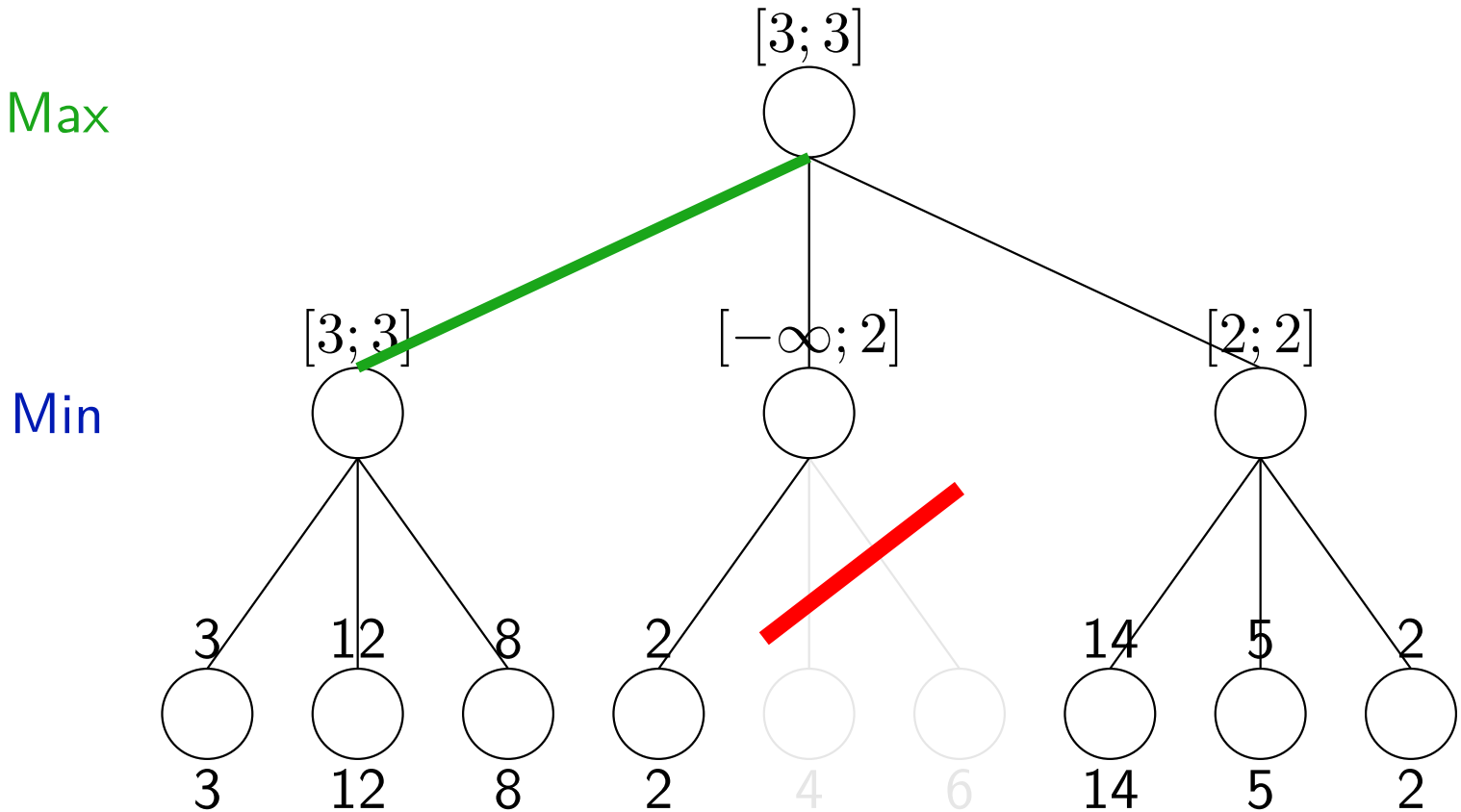

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- empirical simple heuristics often work well
  e. g., ordering 'captures, threats, forward, backward' for chess
  up to factor 2 optimal

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- If feasible 'brute force' is better than 'intelligence'.