ucc
Coláiste na hOllscoile Corcaigh, Éire
University College Cork, Ireland

CS4618 Artificial Intelligence I

Today: More Uninformed Search

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Announcement: Practicals

Remember weekly assignments
discussed in 'practicals'

Fact weekly 'practicals'
each Wednesday, 10–11am
in WGB G21
starting next week

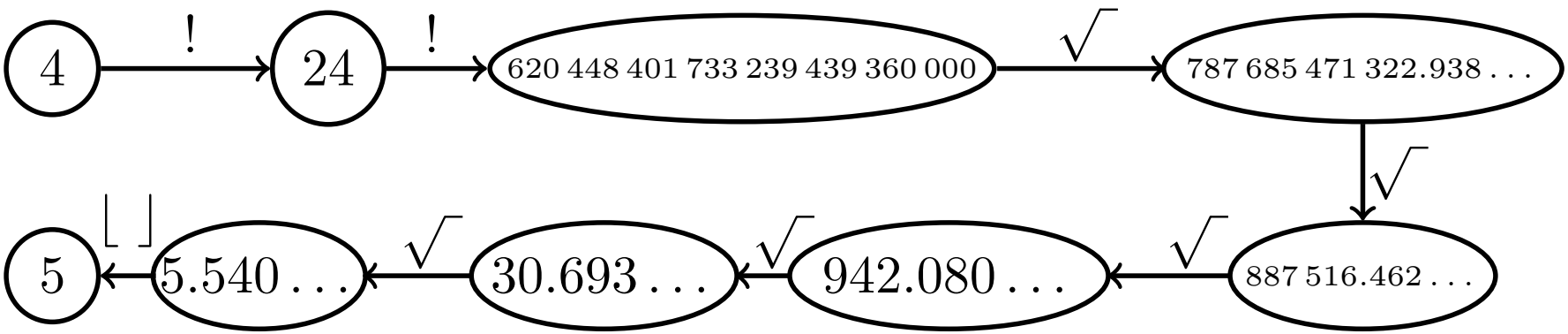
Weekly assignments handed out each Friday
due the next Wednesday
discussed one week later in 'practicals'

Exception today
weekly assignment not to be returned
but to be discussed next Wednesday in 'practicals'

Plans for Today

Knuth's Number Game

Remember initial state 4, goal state 5
actions !, $\sqrt{\ }$, $\lfloor\ \rfloor$; cost 1 each



Cost 8

Uninformed Search

Assume the search algorithm knows

- the initial state,
- possible actions,
- how to find out which actions are applicable,
- how to apply the transition model,
- how to identify a goal state.

Notion blind search or uninformed search

Observation search algorithms can only
determine the order in which states are generated

Breadth-First Search

Idea Expand shallowest node in frontier first.

Breadth-First Search

```
function Breadth-First-Search(problem) returns solution or failure
    node = problem.initial-state
    if problem.is-goal(node.state) then return node.state
    frontier = queue with node as only element
    explored = ∅
    Repeat
        If Empty?(frontier) then return failure
        node = Remove(frontier)
        insert(node, explored)
        for each action in problem.actions(node.state) do
            child = problem.transition-model(node.state, action)
            if problem.is-goal(child.state) then return child.state
            if child ∉ explored und child ∉ frontier
            then insert(child, frontier)
```

Measuring the Performance of Search Algorithms

- Completeness
  Is the algorithm guaranteed to find a solution if one exists?
- Optimality
  Is the algorithm guaranteed to find a solution with minimal path cost if one exists?
- Time Complexity
  How long does the algorithm take to find a solution?
  Remark Often proportional to the number of states generated.
- Space Complexity
  How much memory does the algorithm need to find a solution?

Measuring with respect to what?

- branching factor $\widehat{=}$ maximum number of applicable actions
- depth $\widehat{=}$ minimal number of steps to a goal state
- $m \widehat{=}$ maximum length of path in the state space

Properties of Breadth-First Search

- Completeness?

  Observation complete ✔
- Optimality?

  Observation not necessarily

  Observation optimal if cost is non-decreasing function of depth (in particular for uniform costs)
- Time Complexity?

  Assume uniform number of applicable actions b and solution at depth d

  Let $N =$ number of nodes created

  Observe

  $1 + b + b^2 + \cdots + b^{d-1} < N \leq 1 + b + b^2 + \cdots + b^{d-1} + b^d$

  $(b^d - 1)/(b - 1) < N < b^{d+1}/(b - 1)$
- Space Complexity?

  Assume as for time complexity

  Observe $(b^{d-1} - 1)/(b - 1) < S \leq b^d/(b - 1)$

Time and Space Complexity for Breadth-First Search

Example with branching factor 10, 1000 bytes per node, 0.000001s per node

depth	nodes	time	memory
2	110	.000011s	107KB
4	11110	.0011s	10.6MB
6	10^6	1.1s	1GB
8	10^8	2min	103GB
10	10^{10}	3h	10TB
12	10^{12}	13 days	1PB
14	10^{14}	3.5 years	99PB

Observations

- memory more critical than time
- only problems with solutions at small depth feasible

Uniform-Cost Search

Consider breadth-first search for problem with uniform costs

Observation breadth-first search optimal
since it always expands shallowest node first

Can we have optimal performance for arbitrary cost functions?

Yes if we take cost into account in deciding about ordering
$\rightsquigarrow$ order in queue according to cost ($\rightsquigarrow$priority queue)
and be careful when re-discovering nodes

Uniform-Cost Search Algorithm

Uniform-Cost Search

function Uniform-Cost-Search(problem) returns solution or failure
    node = problem.initial-state
    if problem.is-goal(node.state) then return node.state
    frontier = priority queue with node as only element
    explored = $\emptyset$
    Repeat
        If Empty?(frontier) then return failure
        node = Remove(frontier)
        insert(node, explored)
        for each action in problem.actions(node.state) do
            child = problem.transition-model(node.state, action)
            if problem.is-goal(child.state) then return child.state
            if child $\notin$ explored und child $\notin$ frontier
            then insert(child, frontier) (updating cost if necessary)

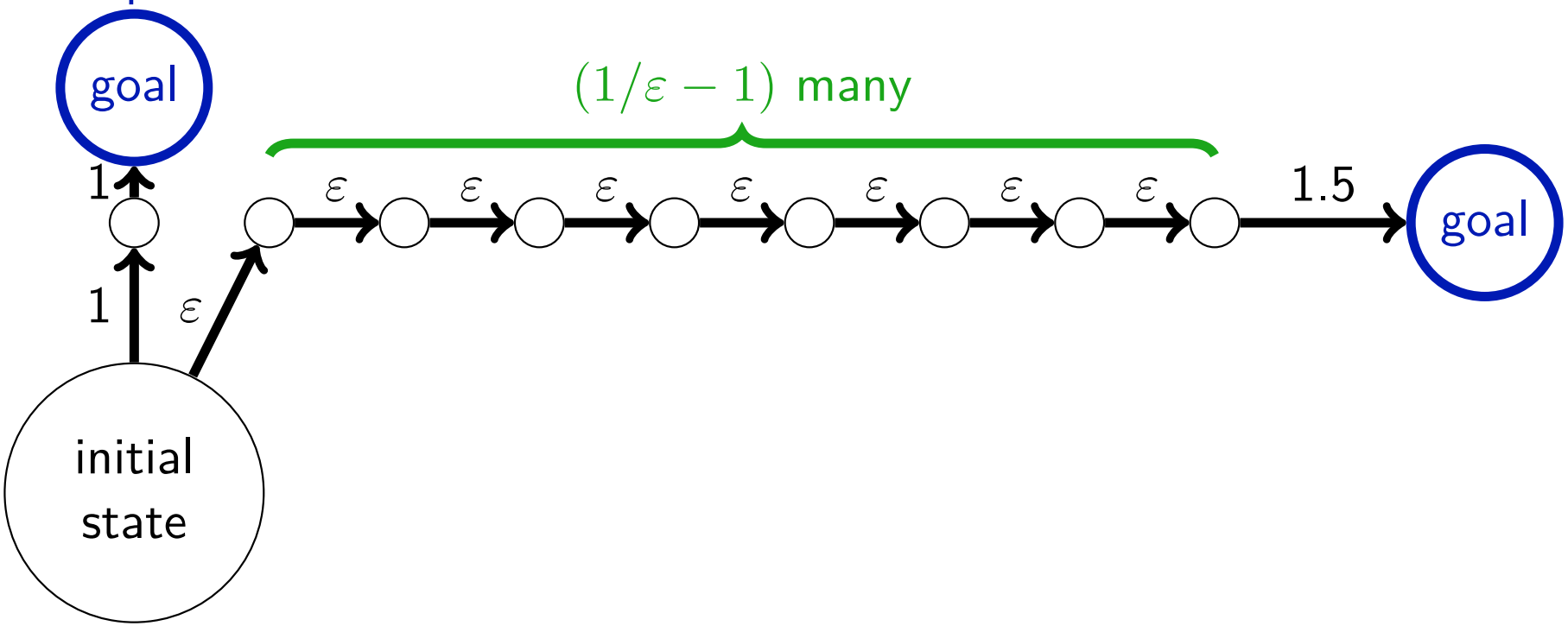
Optimality of Uniform-Cost Search

Key Observation node chosen for expansion has 'correct' cost

Consequence minimal cost solution found first

Observation time and memory requirement unbounded
in comparison to breadth-first search

Example



Depth-First Search

Idea Expand deepest node in frontier first.

Depth-First Search

```
function Depth-First-Search(problem) returns solution or failure
    node = problem.initial-state
    if problem.is-goal(node.state) then return node.state
    frontier = stack with node as only element
    explored = ∅
    Repeat
        If Empty?(frontier) then return failure
        node = Pop(frontier)
        insert(node, explored)
        for each action in problem.actions(node.state) do
            child = problem.transition-model(node.state, action)
            if problem.is-goal(child.state) then return child.state
            if child ∉ explored und child ∉ frontier
            then Push(child, frontier)
```

Comparing Depth-First and Breadth-First Search

- Completeness?

Breadth-First	Depth-First
yes	no (see Knuth's game)

- Optimality?

Breadth-First	Depth-First
in general no	in general no
with uniform cost yes	with uniform cost no

- Time Complexity?

Breadth-First	Depth-First
$\frac{b^d-1}{b-1} < N < \frac{b^{d+1}}{b-1}$	$d \leq N < b^{d+1}$

- Space Complexity?

Breadth-First	Depth-First
$\frac{b^{d-1}-1}{b-1} < S < \frac{b^d}{b-1}$	$b \cdot d \leq S < b \cdot m$

Backtracking

Observation backtracking $\widehat{=}$ modified depth-first search
(made more space efficient)

- generate new states and explore
  instead of generating all and then exploring
- create next state as modification of current state and un-do
  instead of copying or creating new from scratch

Observation

- reduces space complexity to m
- requires capability of modifying and restoring states
- requires capability of creating 'next' state

Depth-Limited Search

Idea limit search to depth l

Remark general Depth-First Search $\widehat{=}$ $l = \infty$

Depth-Limited Search

```
function Depth-Limitied-Search(problem, limit) returns solution or failure/cutoff
    return Recursive-DLS(make-node(problem.initial-state), problem, limit)

function Recursive-DLS(node, problem, limit) returns solution or failure/cutoff
    if problem.is-goal(node.state) then return node.state
    else if limit=0 then return cutoff
    else
        cutoff-occured = false
        foreach action in problem.actions(node.state) do
            child = problem.transition-model(node.state, action)
            result = Recursive-DLS(child, problem, limit − 1)
            if result=cutoff then cutoff-occured=true
            else if result ≠ failure then return result
        if cutoff-occured then return cutoff else return failure
```

On Depth-Limited Search

- Space Complexity
  $\leq b \cdot l$
- Time Complexity
  $O\left(b^l\right)$

Observation cutoff due to reaching the limit
additional source of incompleteness

How can we set the depth limit?

- problem properties need to be taken into account
- useful diameter $\widehat{=}$ largest distance between two states in state space

Iterative Deepening Depth-First Search

Idea apply Depth-Limited Search with minimal feasible depth limit increasing the limit as needed

Iterative Deepening Depth-First Search

```
function Iterative-Deepening-Search(problem)
returns solution or failure/cutoff
    depth = 0
    Repeat
        result = Depth-Limited-Search(problem, depth)
        if result ≠ cutoff then return result
        depth = depth + 1
```

About Iterative Deepening Depth-First Search

Observation 'shallow' levels are explored multiple times

How costly is this obvious wastefulness?

Consider problem with solution at depth d
and uniform branching factor b

Remember BFS and DFS $O\left(b^{d+1}\right)$

$$\underbrace{1}_{\text{level 0 at limit 0}}$$

$$+ \underbrace{1}_{\text{level 0 at limit 1}} + \underbrace{b}_{\text{level 1 at limit 1}}$$

$$+ \underbrace{1}_{\text{level 0 at limit 2}} + \underbrace{b}_{\text{level 1 at limit 2}} + \underbrace{b^2}_{\text{level 2 at limit 2}} + \cdots$$

$$= \sum_{l=0}^{d-1} \underbrace{(d-l)}_{\text{repetitions}} \cdot b^l = \cdots = O\left(b^{d+1}\right)$$

Bidirectional Search

Idea start two searches,
one from the initial state and
one from a goal state and search until the frontiers intersect

Why does this make sense?

Remember time complexity $\approx b^d$ for DFS (and BFS)

Observe both searches stop at depth $d/2$
and $b^{d/2} + b^{d/2} \ll b^d$

Requirements

- goal known
- backwards search feasible (need to be able to 'reverse actions')

Remark 'dummy goal' useful in case of multiple goals

Comparing Algorithms for Uninformed Search

Algorithm	Completeness	Optimality	Time	Space
Breadth-First	yes (for finite b)	yes (for uni. costs)	$\Theta\left(b^d\right)$	$\Theta\left(b^d\right)$
Uniform-Cost	yes (for finite b, cost > 0)	yes	(depends on cost)	(depends on cost)
Depth-First	no	no	$O(b^m)$	$O(b \cdot m)$
Depth-Limited	no	no	$O\left(b^l\right)$	$O(b \cdot l)$
Iterative Deepening	yes (for finite b)	yes (for uni. costs)	$O\left(b^d\right)$	$O(b \cdot d)$
Bidirectional	yes (for finite b, with BFS)	yes (for uni. costs, with BFS)	$O\left(b^{d/2}\right)$	$O\left(b^{d/2}\right)$

Summary & Take Home Message

Things to remember

- breadth-first search
- uniform-cost search
- depth-first search and variants
  - depth-limited search
  - iterative deepening depth-first search
  - bidirectional search

Take Home Message

- Trade-offs between space, time, and other properties make it worthwhile knowing (and applying) different kinds of algorithms for uninformed search.
- DFS is vastly more space-efficient than BFS.
- Even in the restricted scenario of uninformed search clever search algorithms can be implemented.