CS1101: Lecture 23

The Digital Logic Level: Arithmetic Circuits

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Lecture Outline

- Arithmetic Circuits
- Adders
- The Full Adder
- How does the Full Adder Work?
- Arithmetic Logic Units
- A 1-bit ALU
- An 8-bit ALU
- **Reading**: Tanenbaum, Chapter 3, Section 2

Arithmetic Circuits

- Arithmetic circuits are logic circuits which are capable of performing simple arithmetic operations such as addition.
- There are many kinds:
  - Shifters
  - Adders
  - Arithmetic Logic Units
- We will just consider adders - in particular, the half-adder and the full-adder circuits - and their use in building a simple ALU.

Adders

- A hardware circuit for performing addition is an essential part of every CPU;
- Below is a truth-table for 1-bit addition along with a circuit known as the **half adder**;
- There are two outputs present: the sum of the inputs, A and B, and the carry to the next (leftward) position

A	B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

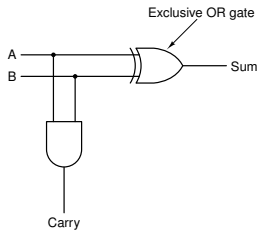


Figure3-17. (a) The truth table for 1-bit addition. (b) A circuit for a half adder.

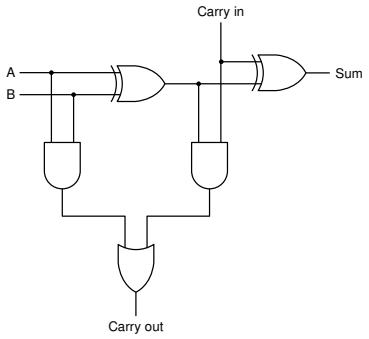
The Full Adder

- A half adder is adequate for summing the low-order bits of two multi-bit input words, but it will not do for a bit position in the middle of the word because it does not handle the carry into the position from the right.
- We need a **full adder**;
- The truth table and circuit for the full adder will be given next.

The Full Adder

A	B	Carry in	Sum	Carry out
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

(a)



(b)

Figure3-17. (a) The truth table for a full adder. (b) A circuit for a full adder.

How does the Full Adder Work?

- From inspection, a full adder is built up from two half adders
- The *Sum output* line is 1 if an odd number of A, B and the *Carry in* are 1
- The *Carry out* is 1 if either A and B are both 1 (left input to the OR gate) ot exactly one of them is 1 and the *Carry in* bit is also 1.
- Together the half adders generate both the sum and carry bits.

Arithmetic Logic Units

- Most computers contain a single circuit for performing the AND, OR and sum of two machine words.
- For n-bit words the circuit is built up of n identical circuits for the individual bit positions.
- We will see a simple example of such a circuit, called an **Arithmetic Logic Unit** or **ALU**.
- It can compute four functions - namely, A AND B, A OR B, $\overline{B}$ or the sum of A and B.
- Which function is computed is determined by whether the **function select** lines F_0 or F_1 contain 00, 01, 10 or 11 (binary).

A 1-bit ALU

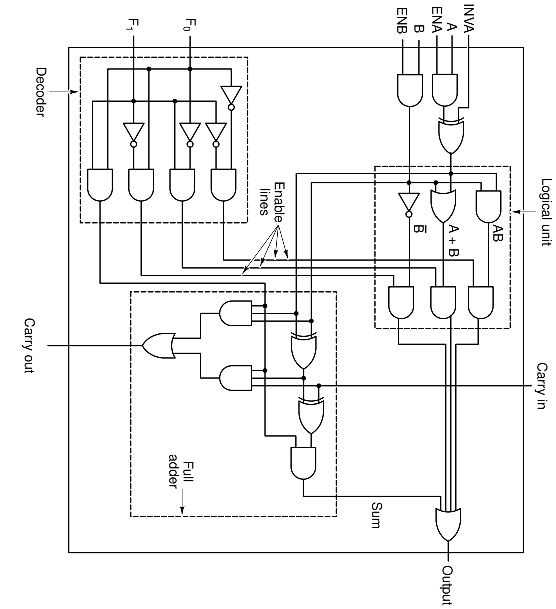


Figure 3-19. A 1-bit ALU.

An 8-bit ALU

- Circuits like the 1-bit ALU shown previously are available as **bit slices**; these allow circuit designers to build ALUs of any desired width.

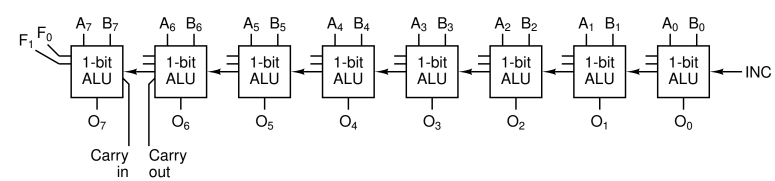


Figure 3-20. Eigth 1-bit ALU slices connected together to make an 8-bit AL