CS1101: Lecture 11
Binary Numbers

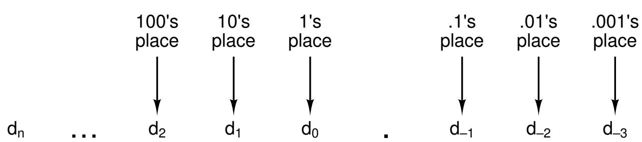
Dr. Barry O'Sullivan
`b.osullivan@cs.ucc.ie`

Course Homepage
`http://www.cs.ucc.ie/~osullb/cs1101`

Lecture Outline

- The General Form of a Decimal Number
- Some Radix Systems
- Examples of Radix Numbers
- Conversions: Binary, Octal, Hexadecimal
- Examples: Conversion
- Decimal-Binary Conversion by Halving
- **Reading**: Tanenbaum, Appendix A.

The General Form of a Decimal Number



$$\text{Number} = \sum_{i=-k}^{n} d_i \times 10^i$$

Figure A.1 The general form of a decimal number

Some Radix Systems

- A radix k number system requires k different symbols to represent the digits 0 to $k-1$.
- Decimal numbers are built up from the 10 decimal digits

$$0123456789$$

- Binary numbers do not use these ten digits. They are all constructed exclusively from the two binary digits

$$01$$

- Octal numbers are built up from the eight octal digits

$$01234567$$

Some Radix Systems

- For hexadecimal numbers, 16 digits are needed. Thus six new symbols are required. It is conventional to use the upper case letters A through F for the six digits following 9. Hexadecimal numbers are then built up from the digits

$$0123456789ABCDEF$$

- Examples of radix numbers:
  - 11
  - 19
  - 7B9
  - 1011011
- To avoid ambiguity, people use a subscript or 2, 8, 10 or 16 to indicate the radix when it is not obvious from the context.
- The representation of numbers may be different when using a different radix.

Examples of Radix Numbers

Binary 1 1 1 1 1 0 1 0 0 0 1

$1 \times 2^{10} + 1 \times 2^{9} + 1 \times 2^{8} + 1 \times 2^{7} + 1 \times 2^{6} + 0 \times 2^{5} + 1 \times 2^{4} + 0 \times 2^{3} + 0 \times 2^{2} + 0 \times 2^{1} + 1 \times 2^{0}$

$1024 + 512 + 256 + 128 + 64 + 0 + 16 + 0 + 0 + 0 + 1$

Octal 3 7 2 1

$3 \times 8^{3} + 7 \times 8^{2} + 2 \times 8^{1} + 1 \times 8^{0}$

$1536 + 448 + 16 + 1$

Decimal 2 0 0 1

$2 \times 10^{3} + 0 \times 10^{2} + 0 \times 10^{1} + 1 \times 10^{0}$

$2000 + 0 + 0 + 1$

Hexadecimal 7 D 1

$7 \times 16^{2} + 13 \times 16^{1} + 1 \times 16^{0}$

$1792 + 208 + 1$

Figure A.2 The number 2001 in binary, octal and hexidecimal.

Conversions: Binary, Octal, Hexadecimal

- Conversion between octal or hexadecimal numbers and binary numbers is easy.
- **To convert a binary number to octal** divide it into groups of 3 bits, with the 3 bits immediately to the left (or right) of the decimal point (often called a **binary point**) forming one group, the 3 bits immediately to their left, another group, and so on.
- Each group of 3 bits can be directly converted to a single octal digit, 0 to 7, according to the conversion given in the first lines of Figure A-3.
- It may be necessary to add one or two leading or trailing zeros to fill out a group to 3 full bits.

Conversions: Binary, Octal, Hexadecimal

- **Conversion from octal to binary** is equally trivial.
- Each octal digit is simply replaced by the equivalent 3-bit binary number.
- **Conversion from hexadecimal to binary** is essentially the same as octal-to-binary except that each hexadecimal digit corresponds to a group of 4 bits instead of 3 bits.

Examples: Conversion

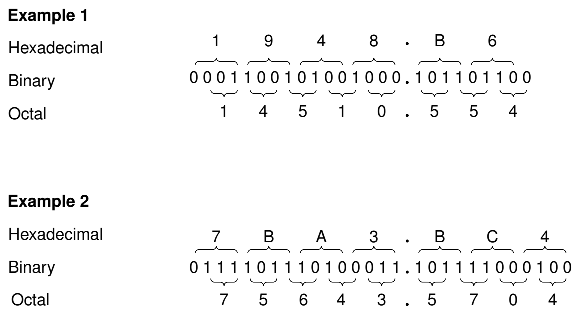


Figure A.4 Examples of octal-to-binary and hexademcimal-binary conversion

Decimal-Binary Conversion

- **Conversion of decimal numbers to binary** can be done in two different ways.
- The first method follows directly from the definition of binary numbers.
- The largest power of 2 smaller than the number is subtracted from the number.
- The process is then repeated on the difference.
- Once the number has been decomposed into powers of 2, the binary number can be assembled with 1s in the bit positions corresponding to powers of 2 used in the decomposition, and Os elsewhere.
- What is the binary representation of the decimal number 20?

$$20_{10} = 16 + 4 = 2^4 + 2^2 = 10100_2$$

Decimal-Binary Conversion by Halving

- This other method (for integers only) consists of dividing the number by 2.
- The quotient is written directly beneath the original number and the remainder, 0 or 1, is written next to the quotient.
- The quotient is then considered and the process repeated until the number 0 has been reached.
- The result of this process will be two columns of numbers, the quotients and the remainders.
- The binary number can now be read directly from the remainder column starting at the bottom.

Decimal-Binary Conversion by Halving

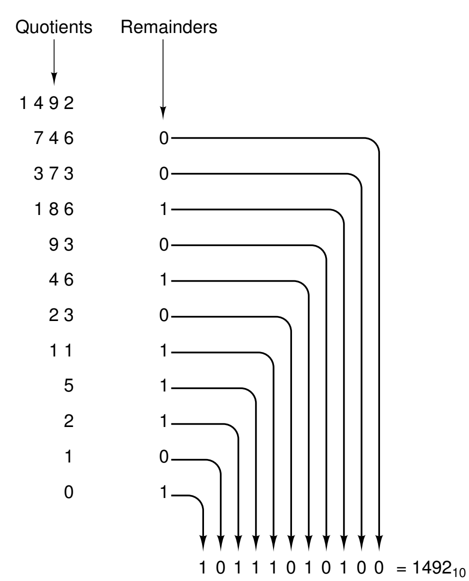


Figure A.5 Conversion of the decimal number 1492 to binary by successive halving.