Anomaly detection for Building Service Components using performance data

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Particle Filter, Gaussian Processes, Maintenance Scheduling

Abstract

The efficient operation of building systems is important energy efficiency, comfort and safety. Determining when maintenance is required or when a fault has occurred is the focus of this work. We show how to use available performance data in a methodology for improved maintenance scheduling through anomaly detection. We apply two statistical prognostic techniques – Particle Filters and Gaussian processes – to sensed data from two HVAC components to illustrate the methodology. We demonstrate that both methods identify occasions when maintenance should be carried out for one of the components, but are less successful on the second component.

1. Introduction

One of the main financial burdens of a building during its life cycle is the cost of maintenance. There are many different methods for scheduling maintenance activities, including reactive, planned, and condition-based maintenance. These methods make a trade-off between equipment health, cost and user-comfort. In industrial applications, a greater importance is placed on maintenance, due to high costs associated with equipment down-time, and so techniques such as condition-based maintenance are commonly used. For the most part, office or educational facilities are not subjected to such stringent maintenance controls, but many of these buildings already have the data needed to enable such maintenance activities. This data is provided from Building Management Systems (BMS) and can be used in conjunction with other tools to manage maintenance activities more effectively.

There are a number of different types of maintenance which are used at present. They are: Routine Maintenance; Emergency Maintenance; Corrective Maintenance; Testing or Failure-finding; Predictive Maintenance; and Performance Based Maintenance (PBM). We consider Performance-based maintenance in this paper. PBM can be further classified as a meeting point between scheduled and reactive maintenance. It allows for faults which were not foreseen at the time of scheduling to be dealt with before they reach failure, i.e. before reactive maintenance is required. Cost allocation can be carried out for performance based but not for reactive maintenance.

In order to implement PBM for building service components, it is necessary to utilise techniques from the area of prognostics. These prognostic techniques can be used to monitor and track the performance of a component and also to predict the future behaviour, within the limitations of the available data with which the technique is trained.

In general there are three main requirements to implement prognostics, (Greitzer & Ferryman 2001) define them as: hardware and sensor technologies; analytically effective predictive methods; and organisational changes to capture the operational, maintenance and logistical benefits made possible by effective prognostic information. Given the predominant occurrence of BMS for controlling and operating building service equipment, there is now a readily available source of hardware and sensor technologies available in many buildings. Also due to the increasing use of Building Information Modelling in building construction, information to support analytical predictive methods can be more easily, accessed.

In this research, we focus on data driven approaches, in the area of prognostics of monitoring and reasoning about parameters. As well as identifying precursors to failure, we also aim to be able to identify reductions in performance, through tracking of performance parameters of components. We apply two advanced techniques for tracking the performance of HVAC equipment: a Gaussian process model and a dynamic model/particle filtering approach. These approaches allow detection of major changes in equipment performance including drift and change points due to faults. We apply the techniques to two real datasets containing sensed data from two different HVAC components, and we demonstrate that the techniques can successfully monitor their performance and identify points in time when a change is occurring, be that a change due to control strategy, load levels or conditions, due to suboptimal operating conditions or a maintenance requirement.

2. Technical Background

(Si et al. 2011) divides statistical prognostic techniques firstly into two segments based on the type of data available, direct condition monitoring data or indirect condition monitoring data. They focus on the area of prognostics which deals with estimating the Remaining Useful Life (RUL) of a component. Within direct CM data, Si highlights the following techniques: regression based, wiener process, gamma process, and markovian-based. The techniques within indirect CM data include stochastic filtering based, covariate-based hazard, and HMM and HSMM based. For this paper, we utilise indirect CM data and apply a Particle filter (section 3.4) and a Gaussian Process approach (section 3.5).

2.1 Particle Filter Prognostics

The first technique employed is a Particle Filter/state space approach (PF). This assumes an underlying state space model in which the states are the efficiency and rate of change of efficiency. The states are observed using a non-linear observation equation which expresses the relation between the input *x* and the output *y* as:

$$y = F(\mathrm{x}, \theta) \tag{1}$$

As this Equation is non-linear, a particle filter is required to track the change in parameters over time. Note that the observation equation is the function relating efficiency to input; i.e. it is the change in the system we are interested in. There are several types of particle filter; the specific PF used here is a sequential importance sampling PF, (Arulampalam et al. 2002) and is detailed in Algorithm 1. A particle filter essentially involves a collection of parameter estimates which are called *particles* denoted, p_k^i the i_{th} particle at time *k*. There are N_s such particles (typically 100) and these are weighted according to how well they match the observations, y_k, by weights, w_{ki}. Those particles that produce estimates close to the observed outputs are given a higher weighting. It becomes necessary to clear out those particles with very low weights after several time steps and re-sample new ones; this is known as the particle degeneracy problem. Finally a smoothed estimate of the process, hat y_k, is obtained by a weighted average of the individual particle estimates. As described by (Li et al. 2006), a '*standard particle filter is developed based on Bayesian sequential estimation'*, where the hidden state of target and its observation at a time t, are denoted by x_t and y_t and the filtering distribution $p(x_t, Y_t)$ stands for the distribution of the target state given all observations $Y_t = (y_1, ..., y_t)$. The Particle Filter (PF) approach utilises state space models. The state-space approach to time-series modelling puts attention on the state vector of a system. The state vector contains all relevant information required to describe the system under investigation (Arulampalam et al. 2002). The state space approach is convenient for handling multivariate data and non-linear/non-Gaussian processes, and it provides a significant advantage over traditional time-series techniques for these problems (Arulampalam et al. 2002). Resampling is necessary to avoid the problem of degeneracy. According to (Arulampalam et al. 2002), degeneracy is the "phenomenon, where after a few iterations, all but one particle will have negligible weight" and a suitable measure of this degeneracy is N_{eff}, the effective sample size.

There is a large amount of literature in the prognostic field on the utilisation of PFs for the estimation of Remaining Useful Life (RUL). These research works focus on identifying a particular feature of the system and utilising this feature to infer the state of the system at a particular point in time and also predict the future state.

For example, (Caesarendra et al. 2010) and (Lall et al. 2011) use PF to indicate the degradation condition of a machine through monitoring vibration levels of a machine. Also, PF has been used in conjunction with other techniques. For example, in (Chen et al. 2011), a high order PF is used in combination with ANFIS (Adaptive neuro fuzzy inference system) to predict the remaining useful life of a cracked carrier plate and a faulty bearing. PF can also be applied in model based approaches, e.g. (Daigle & Goebel 2009) in order to estimate the damage for a pneumatic valve.

2.2 Gaussian Processes for Prognostics

A Gaussian Process (GP) represents observations from a process as draws from a jointly multivariate normally distributed as:

$$y \sim N[\mu_x, C_{x,x}] \tag{2}$$

where ~ is used to denote drawn from, *N* denotes a Gaussian distribution, *y* is a vector of observations, *x* is a vector of sample times. μ_x is the mean of the process at the sample times and $C_{x,x}$ is the covariance matrix. The covariance between two points is defined by a kernel which is often referred to as the covariance function. The covariance function is central to a GP model and defines the structure of the model. In order to be a valid covariance matrix we require that $C_{x,x}$ be positive definite. However, the sum or product of two valid covariance functions is also a valid covariance function allowing us to tailor the covariance function to the particular dataset being analysed. For the GP's used in this paper the covariance function is derived from the Matérn kernel which defines the covariance between two points as:

$$C_2(h) = \frac{1}{\Gamma(\nu)2^{\nu}} \left(\frac{2\sqrt{\nu}\,|h|}{\theta}\right)^{\nu} K_{\nu}\left(\frac{2\sqrt{\nu}\,|h|}{\theta}\right) \quad (3)$$

where h is the separation of the input points, K_{ν} is the modified Bessel function, θ and ν are parameters of the kernel with θ controlling the scale and v the shape of the kernel, Γ is the Gamma function. This kernel is chosen as it allows for a wide variety of kernel shapes with the use of only 2 parameters. Later we will use the fact that the sum or product of two valid covariance functions is also a valid covariance function. This allows us to tailor the covariance function to the particular dataset being analysed.

Finally, both the covariance function and data scaling parameters need to be estimated. For this work, and as described by (Brahim-Belhouari & Bermak 2004), these parameters are estimated by maximising the log likelihood function with respect to [α, β, σ_b^2, Υ, σ_r^2]. This function, also known as the log marginal likelihood, may be expressed as (Rasmussen & Williams 2006):

$$L = -\frac{1}{2} y^T (C_{x,x} + \zeta_x)^{-1} y - \frac{1}{2} \log|(C_{x,x} + \zeta_x)| - \frac{n}{2} \log(2\pi) \quad (4)$$

There are many examples in the literature of the use of GP for prognostics. (Goebel et al. 2008) utilised GP regression to estimate end of life for batteries where the shape and position of EIS plots are used as diagnostic features in the GPR. (Mohanty et al. 2007) investigates GPs for use with a hybrid model of fatigue crack growth in metal alloys with a physics-based state space based model. (Boskoski et al 2012) uses GP to estimate the RUL for faulty bearings.

Also with respect to timeseries GP, (Wang et al 2006) and (Kocijan & Tanko 2011) uses a GP timeseries models to track human motion capture data and to describe gear health respectively. (Kocijan & Tanko 2011) use 2 covariance functions, the sum of the Matérn and polynomial covariance function and the neural network covariance function and (Mohanty et al. 2007) also implements 2 types of covariance functions, a radial based anisotropic and a neural network based isotropic. GPs can also be integrated with other techniques, e.g. (Dong & He 2007) combines GP with Hidden semi Markov Model.

3. *Case Study and Methodology*

3.1 Gaussian Processes for Prognostics

One sample data set used here is from a water to water heat pump located in the Environmental Research Institute, University College Cork. This particular heat pump is used to provide hot water to an underfloor heating system. It is served by a vertical, open-loop piping system connected to an underground aquifer located on site. 4 years of data are available for this component including, the power into HP01 in kWh, heat output, kWh, temperature of the aquifer, degree Celsius, temperature on the condenser side, degree Celsius, and temperature on the evaporator side, degree Celsius.

In order to track the performance of the Heat Pump, the Coefficient of Performance (COP) is calculated using the formula:

COP = Heat Output/ Power Input (5)

There are known relationships between (1) the COP and the temperature difference between the evaporator and condenser and (2) the COP and the temperature of the water entering the heat pump on the aquifer side. For these analyses, the first relationship is chosen, in order to negate somewhat the effect of seasonal variances on the results.

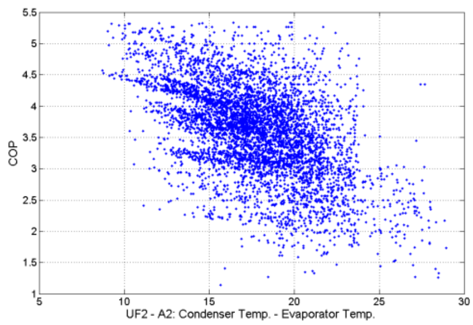

Figure 1: Original Data for Heat Pump COP vs A2

3.2 Case Study: Heat Exchanger HE01

The Heat Exchanger used in this example is a plate heat exchanger. It is located in the Environmental Research Institute, University College Cork. Its main purpose is to transfer heat from a cooling circuit to a geothermal heating circuit. There are a number of variables measured with respect to HE01. The most useful for this example are the temperatures into,C2, and out of, C1, HE01 on the cooling circuit side and the temperature into HE01 on the aquifer side, A1. A2, the temperature out of HE02, will also be used, see Figure 2. It is known that HE01 underwent an overhaul in October 2009, and so it is assumed that the condition of HE01 in October 2009 is 100%. The measure used to monitor the performance of HE01 is the plot of the temperature difference between the fluid in on the hot side and out on the cold side versus the temperature difference between the fluid out on the hot side and in on the cold side, due to counterflow principles, Figure 3.

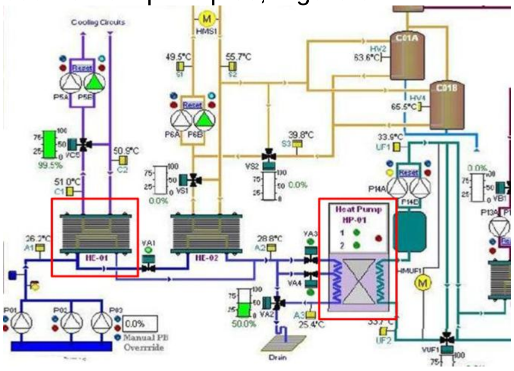

Figure 2: ERI HVAC schema

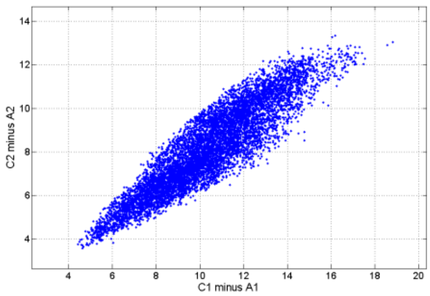

Figure 3: HE01 performance indicator, raw data

3.3 General Methodology

The aim of this paper is to provide a way for the facility manager to make decisions about maintenance actions, based on data, such that can be seen in Figure 1 and Figure 3. In order to do this effectively, a number of steps are followed. (1) Performance indicators are identified: Specific relationships between measured data are investigated; Acceptable upper and lower bounds for these performance indicators are defined. (2) Performance indicators are extracted and tracking using: Particle Filters; Gaussian Processes. (3) Tracked performance indicators are evaluated against predefined acceptable ranges, for the case of the PF, and evaluated against an optimal scenario, in the case of the GP. For this paper, we assume the ranges, due to knowledge of when the components are at 100% efficiency.

3.3.1 Particle Filter

The sequence of the particle filter used here is described by Algorithm 1. It details the steps necessary to use the particle filter methodology for the available HVAC data. The algorithm can be divided into 4 steps: Initialisation of the variables; Propagation of the particles, weighted probabilities calculated and normalised; if necessary, resampling; and, posterior value for y_{obs} is calculated. In Algorithm 1, p_0 are the initial estimates or initial set of particles to feed into the particle filter. They are found through trial and error, by using regression technique, such as *robustfit* and *polyfit* in Matlab, and by using an optimisation technique in conjunction with the process equation for the state space model. N_s represents the number of particles per iteration and T is the number of iterations. θ_h is the set of resultant particles and w is the set of applied weights. The symbol $\mathcal{N}$ represents a normal probability density function.

Data: y^k, x^k, Ns, T, p_0
Result: $\theta h[1,T]$.
begin
Initialisation: $Pt_i = \mathcal{N}(0, 0.01)$;
$w^{(0)}_{i=1:Ns} = 1/Ns$;
$\tilde{y}^{(0)}_i = \sigma_{par\_0}$;
$\theta^{(0)}_i = par\_0$;
Prop: **for** $k = 2$ *to* T **do**
for $i = 1 \rightarrow Ns$ **do**
Prop: $\theta^k_i = \theta^{k-1}_i + Pt_(i)$;
$\hat{y}^k_i = \mathcal{F}(\theta^k_i, x^k)$;
$\tilde{y}^k_i = y^k - \hat{y}^k_i$;
$\delta_i \sim \mathcal{N}(\tilde{y}^k_i, 0, \theta_{\tilde{y}^{k-1}})$;
$wk^k_i = wk^{k-1}_i * \delta_i$;
end
Norm: $w^k_i == \frac{w^k_i}{\sum_{i=1}^{Ns} w^k_i}$;
$N_{eff} = \sum_{i=1}^{Ns}(w^k_i)^2$;
Resamp: **if** $N^k_{eff} < (0.5 * Ns)$ **then**
Set $\theta^k_{i=1:N} \longrightarrow \theta^k_{j=1:N} with P_j \propto w_j$;
$w^k_{i=1:N} = 1/Ns$;
end
$\theta h^k = 0$;
for $i = 1$ *to* Ns **do**
$\theta h^k = \theta h^k + \theta^k_i * w^k_i$;
end
end
end

Algorithm 1: Generic Particle Filter Algorithm

3.3.2 Gaussian Processes

In this research, 2 GPs were applied to the case study dataset: (1) GP with 1 covariance function; (2) GP yearly with 2 covariance functions.

The first GP utilises one covariance function, derived from a Matérn kernel. The methodology involves scaling the time variable and so the kernel models the relationship between the output and the non-time related inputs. Also, while evaluating the GP with one covariance function, two methods of estimation of the function and kernel parameters were evaluated. These two methods were: (a) including the effect of each data point on the maximum likelihood in the estimation; and (b) calculating one maximum likelihood estimation value. It was decided from this evaluation to utilise the second method, calculation of one maximum likelihood estimation value, as the resulting GP tracked the parameters with a much lower Root Mean Squared Error (RMSE). For the second GP, two covariance functions were involved in the calculation of the final kernel. Both of the kernels were chosen to be Matérn kernels. These two kernels were added as it was decided that a lower value in one kernel should not predict a low overall value.

4. Results

This section will introduce the case study dataset and present the results from the particle filtering algorithm and the various GP techniques.

4.1 Particle Filter Results

4.1.1 Heat Pump

For the heat pump data, the relationship between the COP of the HP and the Temperature difference across the heat pump is used to generate parameters to monitor the condition of equipment. The equation used for the state space model process equation is:

$$Y = \alpha * x^{\beta} \quad (6)$$

where y is the COP and x is (UF2-A2) and the relationship is represented by F within the general algorithm (see Algorithm 1) and Figure 1 illustrates this relationship. The resulting tracked parameters from the PF can be seen in Figure 4 and Figure 5. It is known that at the beginning of the heating season in 2009, the refrigerant in HP01 was topped up and so the working pressure increased. It can be clearly seen that this had a large effect on the efficiency of the heat pump, as both parameter 1 and 2 increase by 0.1 during this period. It can also be seen that the yearly mean decreases slightly consistently from 2009 to 2011.

4.1.2 Heat Exchanger

For the heat exchanger data, the relationship between the temperature differences at the inlets and outlets is used to model the process equation. It is of the form:

$$y = \alpha + \beta x \quad (7)$$

where *y* is C2minusA2 and *x* is C1minusA1, these are the differences between the inlet/outlet temperatures on the heating and cooling side.

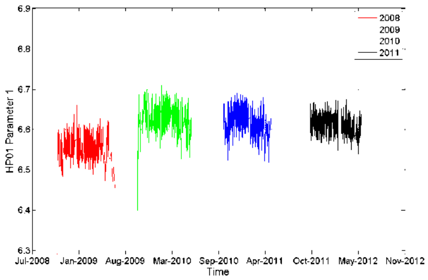

Figure 4: Parameter 1 from HP01 PF results

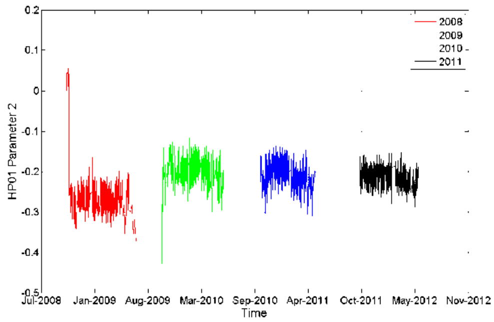

Figure 5: Parameter 1 and 2 from HP01 PF results

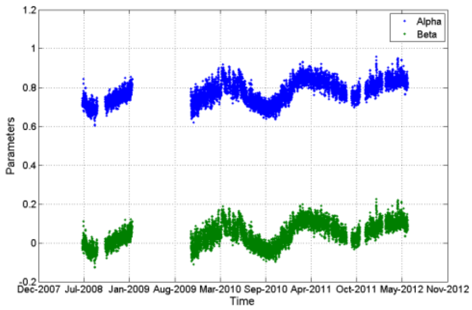

Figure 6: PF parameters for HE01

Note: there is a period of data missing from the second half of the first year. The obvious trends in the two parameters can be seen as a gradually heightening of the parameters as the years progress. Due to the nature of heat exchanger failure, we can assume here that this represents a gradual build-up of deposits in the heat exchanger.

4.2 Gaussian Processes

4.2.1 GP – yearly, 1 covariance function

For this model, one covariance function is used and the general principle is as described in section 2.2. The root mean squared error for these processes can be found in Table 1 and Table 2. As before, the estimated kernels were then graphed for each data segment under a constant time and temperature ranges (Figure 7).

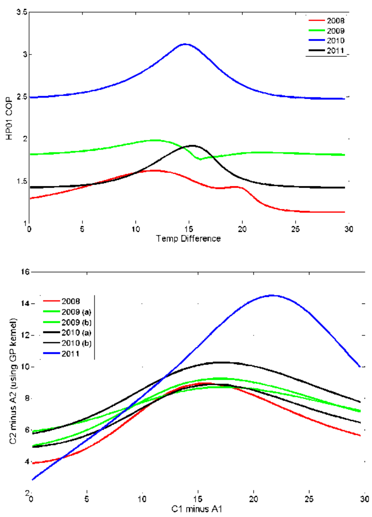

Figure 7: Kernel parameters evaluated for possible range of Δt values for HE01and HP01

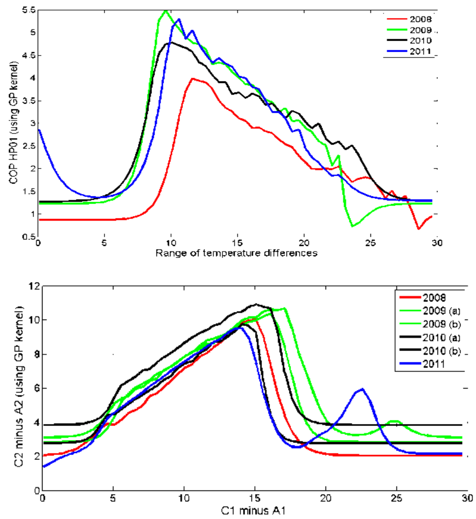

Figure 8: Kernel evaluated for possible range of Δtemp values at mean time value for each year, HE01 and HP01

4.2.2 GP – yearly, 2 covariance functions

In order to account for the influence of time on the relationship between the COP and the difference in temperature across the heat pump, and the comparative relationship for the HE01 data, two covariance functions are used instead of one. For both the time and the temperature difference relationships, a Matérn kernel is used. Again, the estimated kernels were then graphed for each data segment at the mean time for each year and for a constant range of temperature, see Figure 8. As stated in section 2.2, it is valid to add or multiply kernels. Both were investigated for this research, and it was decided that addition was the more suitable combination mechanism. The justification for this is that addition allows for either one of the kernels to attribute a high COP value whereas multiplication would constrain the results so that in any combination with one poor and one high COP reading, the poor reading would override the high value.

4.3 Tracking Ability

With regard to tracking ability, both the particle filter and the Gaussian processes performed to the same level for the HP01 dataset, all RMSE were within 0.45 to 0.55. With the GP 2 covariance functions outperforming slightly the GP with one covariance function.

Year	GP 2 cov	GP 1 cov	PF
2008	0.5273	0.5319	0.515
2009	0.5207	0.5296	0.515
2010	0.4977	0.5303	0.515
2011	0.4891	0.5403	0.515

Table 1: RMSE for GP results for HP01

For the HE01 dataset, the particle filter out performs both GP processes. Also, the results for the GP processes are quite variable over the 4 year period. This may be due to the oscillatory nature of the relationship between the temperature differences across the heat exchanger.

Year	GP 2 cov	GP 1 cov	PF
2008	0.4076	0.5404	0.4573
2009	0.6112	0.7195	0.4573
2010	0.2931	0.4343	0.4573
2011	1.2823	0.9058	0.4573

Table 2: RMSE for GP results for HE01

The resulting RMSEs give credibility to the tracking performance of both techniques and so we can utilise them in future work to predict what the performance of the components will be.

When the two GPs are compared, it can be seen that the GP with 2 covariance functions outperforms that with only 1 function, with respect to RMSE but also with respect to what we would expect, i.e. we know that for both datasets 2011 was a poorer performing year than 2010, yet with only 1 covariance function this is not picked up.

4.4 Maintenance Planning from Tracked Parameters

This section will illustrate how these tracked parameters can be utilised to indicate if maintenance is required for a component. As we apply two statistical techniques in this paper to track the underlying parameters of the HVAC components, two slightly different techniques are utilised to apply the allowable ranges to the datasets for each statistical technique.

4.4.1 Particle Filter

For the particle filter parameter results, before allowable ranges could be set, it was first necessary to regress the parameters with respect to influencing factors. For HP01, A1, the temperature of the Aquifer water is used. This is to reduce the effect of seasonal changes on the data. Likewise, for HE01, the parameters are regressed against A1. For both datasets, the slope and intercept of the regressed data are compared against preset ranges. It was found for the HP01 dataset, that the intercept was most critical for parameter 1 and that the slope was critical for parameter 2. While for the HE01 dataset, the slope was most critical for both parameter 1 and 2.

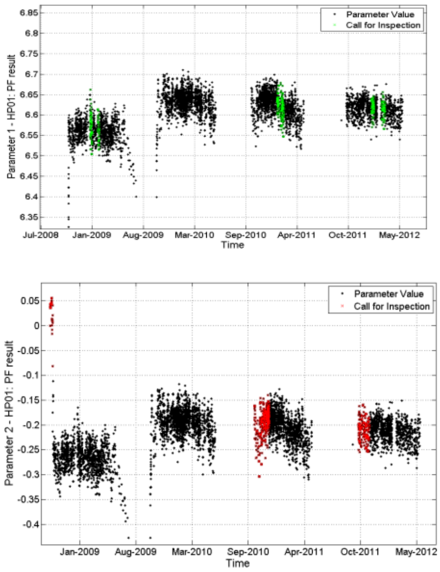

Figure 9: Parameter 1 and 2 for HP01 with inspection points

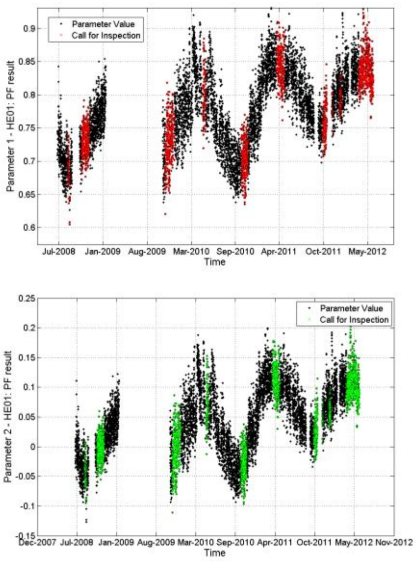

Figure 10: Parameter 1 and 2 for HE01 with inspection points

Figure 9 and Figure 10 illustrates the predicted inspection calls when the chosen ranges are applied to the heat pump and heat exchanger respectively. The optimum parameters are taken to occur at the beginning of October 2009, for both the heat exchanger and the heat pump datasets. The range for parameter 1 worked reasonably well for the heat pump data, the known year where the heat pump was performing sub optimally, 2008, has a number of inspection calls. Also, in 2010 and 2011 a number of inspections are highlighted. The 2010 results are questionable, but it can be seen that in 2011 the performance of the heat pump is decreasing. While parameter 2 did not perform as well, it did highlight a number of inspection points at the beginning of 2008. For the heat exchanger, this method is affected by the seasonal variance in the data. More work is needed to isolate the appropriate ranges to indicate when maintenance is required.

4.4.2 Gaussian Processes

For the GP results a different scenario is implemented to determine when maintenance should be scheduled. The difference between the output at 100% component health and that at the current time is tracked and this can act as a guide for the facility manager to decide when the cost is great enough for maintenance to be warranted.

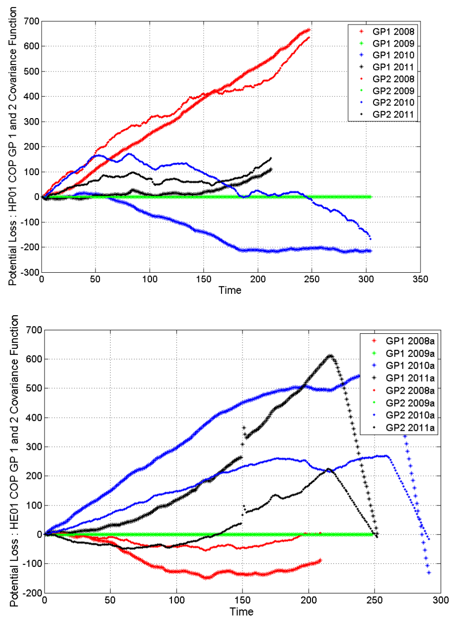

Figure 11: HE01 and HP01 GP 1 and 2 - Drop in output with respect to optimum output

Figure 11 illustrates the optimal output (2009 – green) and the cumulative sum of each year minus the optimal output. This method works well for the heat pump. It can be clearly seen that 2009, 2010 are performing well while, 2008 and 2011 are not. As with the PF ranges, this method is not performing well on the heat exchanger data. The results replicate the GP tracking results: discrepancies in tracking are also evident here.

5. Conclusion

In conclusion, a methodology was presented in this paper to aid a facility manager in deciding whether to schedule maintenance based on the performance of a component. Two statistical techniques are utilised and both were found to perform well on one dataset, in terms of tracking ability and inspection requirement detection. Given the nature of the second dataset, it can be concluded that it is necessary to utilise data in this method which is not influenced by seasonal factors.

6. Future Work

The work presented here tracked performance parameters for HVAC equipment. The prediction of these parameters is still required to complement the methodology. Also, in order to evaluate these two chosen techniques on a wider scale, they should be test with large dataset of anomaly data. Also isolating better ranges for the parameters is a very important area of this research and in the future, it is envisaged to evaluate the available techniques to do so. In order to effective, more datasets with anomalies present are required. Finally, it is necessary to provide a means to ensure no seasonal variations are present in the data.

Acknowledgments

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