

Electrical Circuits and Components

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January 23, 2013

1 Resistors

Resistance is provided in electrical circuits by resistors. The leads of a resistor can be axial or radial. Traditionally, resistor values are colour coded. There are two systems: Body-Tip-Spot and End-to-Centre. The systems only influence the order in which the values are read, not the actual colour codes.

Colour	Number
Black	0
Brown	1
Red	2
Orange	3
Yellow	4
Green	5
Blue	6
Violet	7
Grey	8
White	9

Table 1: Resistor colour codes

These values are read in order from end to centre or in the order body-tip-spot. The first value gives the first significant digit, the second value gives the second significant digit and the third value gives a multiplier specifying a power of ten (or, more simply, the number of zeros). There are several mnemonics for remembering the resistor colour code. One of these is “Bye-Bye Rosie, On You Go to Birmingham Via Great Western”.

Colour	Tolerance
Gold	5%
Silver	10%
No Colour	20%
Other	% as for code

Table 2: Resistor tolerance codes

1.1 Examples

Red-Orange-Black = 2-3-No zeros = 23Ω

Brown-Green-Black = 1-5-No zeros = 15Ω

Green-Red-Orange = 5-2-000 = $52k\Omega$

Red-Green-Yellow = 2-5-0000 = $250k\Omega$

Orange-Orange-Orange = 3-3-000 = $33k\Omega$

1.2 Ohm's Law

The current that flows in an electric circuit is directly proportional to the emf (voltage) applied and inversely proportional to the resistance of the circuit.

$$I = E/R$$

From this, we get $R = E/I$ and $E = IR$

In these equations, current is always expressed in Amperes (A), emf in Volts (V) and resistance in Ohms (Ω).

1.2.1 Examples

1.

A device is connected to the terminals of a battery. By measurement, the voltage across the ends of the wire is 12V and the current flowing in the circuit is 3A. What is the resistance of the device?

Answer: $R = E/I = 12/3 = 4$

2.

What is the voltage across a 25Ω resistor when a current of 200mA is flowing through it?

Answer: $E = IR = 25 \cdot 0.2 = 5V$

3. A voltage drop of 60V is measured across a resistance of $12k\Omega$. What current is flowing?

Answer: $I = E/R = 60/12000 = 5\text{mA}$

4.

If the resistance in 3. above is provided by a resistor with bands coloured Brown- Red-Orange and Silver, what are the limits of the current that may be flowing through the resistor?

Answer (a) Lower Limit: $R = 12000 + 10\% = 13200\Omega$. $I = E/R = 60/13200 = 4.5\text{mA}$ approximately.

(b) Upper Limit: $R = 12000 - 10\% = 10800\Omega$. $I = E/R = 60/10800 = 5.5\text{mA}$ approximately.

1.2.2 Exercises

1.

A bulb from your car headlamp has a rating of 12V and 4A. What is its resistance?

2.

An electric kettle draws 2.5A from the 240V mains supply when it is switched on. What is the resistance of its element?

3.

What value of resistance is required to limit a current driven by a voltage source of 12V to a flow of 6mA?

2 The Series Circuit

When resistors are connected in series, the total resistance equals the sum of the individual resistances.

$$R_t = R_1 + R_2 + R_3 + \dots + R_n$$

The same current flows through every resistance.

2.1 Kirchoff's Second Law

The sum of the voltage drops across series resistances in a closed circuit equals the total voltage applied to the circuit.

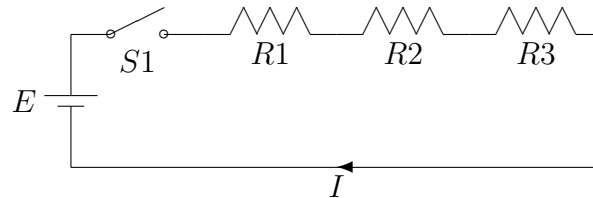


Figure 1: Resistors in series

2.2 Example

A closed circuit powered by a 12V battery consists of three resistors with values of 2Ω , 4Ω and 6Ω connected in series. What is the voltage drop across each resistor?

Consider what you know:

$$R_t = R_1 + R_2 + R_3$$

$$I_t = I_1 = I_2 = I_3$$

$$V_t = V_1 + V_2 + V_3$$

So an equivalent circuit comprises a resistance of 12 ohms connected across a 12V source.

Thus the current flowing is $I = E/R = 12/12 = 1\text{A}$

The voltage drops across the individual resistors are given by

$$V_1 = IR_1 = 1 \times 2 = 2\text{V}$$

$$V_2 = IR_2 = 1 \times 4 = 4\text{V}$$

$$V_3 = IR_3 = 1 \times 6 = 6\text{V}$$

It can be seen that the sum of the voltage drops equals the applied voltage.

2.3 Example

A circuit contains three resistors connected in series across a 100V source. The current flow is 2A. Resistors R_1 and R_2 have known values of 5Ω and 10Ω respectively.

Calculate (i) the resistance of the entire circuit, (ii) the value of resistor R_3 and (iii) the voltage drop across each of the three resistors.

Step 1

Sketch the circuit. Fill in the known values and leave the unknown values blank.

Thus,
 $I_t = 2A$
 $V_s = 100V$
 $R_t = ?$

$I_1 = I_t = 2A$
 $V_1 = ?$
 $R_1 = 5\Omega$

$I_2 = I_t = 2A$
 $V_2 = ?$
 $R_2 = 10\Omega$

$I_3 = I_t = 2A$
 $V_3 = ?$
 $R_3 = ?$

Step 2

$I_t = 2A$
 $V_s = 100V$
 $R_t = ?$

So $R_t = V_s/I_t = 100/2 = 50\Omega$

$R_t = R_1 + R_2 + R_3$
 $50 = 5 + 10 + R_3$
 $R_3 = 50 - 15 = 35\Omega$

$V_1 = I_t R_1 = 2 \cdot 5 = 10V$
 $V_2 = I_t R_2 = 2 \cdot 10 = 20V$
 $V_3 = I_t R_3 = 2 \cdot 35 = 70V$

2.4 Exercises

1.

A circuit branch consists of three resistors, R_1 , R_2 and R_3 , with values of 10, 20 and 60 respectively, connected in series between points A and B. Due to the tight design of the equipment chassis, you can only get your voltmeter probes across R_1 , where you measure a voltage drop of 5V. Calculate the voltage drops across R_2 and R_3 . Hence give the total voltage applied across

points A-B.

2.

You have received a present of a car spotlight rated at 2A, designed to work with a 6V battery. However, your car has a 12V battery. What value of resistance must you connect in series with the spotlight before you can safely switch it on?

3 Potential Divider

A potential divider chain is a circuit designed to provide current at a reduced voltage. Input voltage is divided between 2 resistors, R1 and R2. Output voltage is taken across R2. Assume the input voltage is V_{in} , output voltage is V_{out} and a current I flows in the circuit.

Then the voltage drop across R1 is IR_1 , voltage drop across R2 is IR_2 and total voltage across the resistors is $I(R_1+R_2)$.

So, $V_{in} = I(R_1+R_2)$, $V_{out} = IR_2$ and $V_{out}/V_{in} = R_2/(R_1+R_2)$ or $V_{out} = V_{in}R_2/(R_1+R_2)$

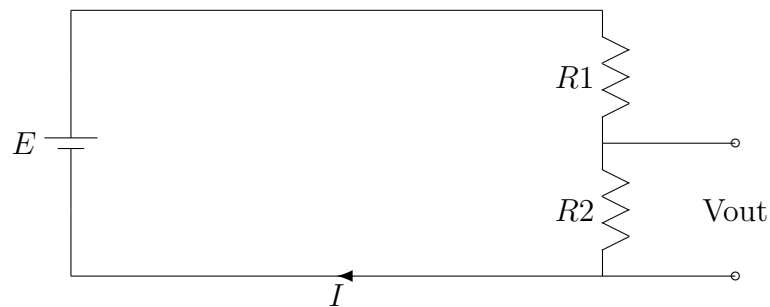


Figure 2: Potential divider

3.1 Kirchoff's First Law

The sum of all the currents flowing towards a junction always equals the sum of all the currents flowing away from the junction.

To use Kirchoff's First Law on a circuit:

- Draw the circuit

- Show the current flow through every resistance in the circuit
- Determine which currents flow towards or away from every junction in the circuit
- The value and direction of unknown currents can often be determined by applying Kirchoff's First Law

3.2 Example

Given the following circuit, determine the currents I1, I4, I6 and I7.

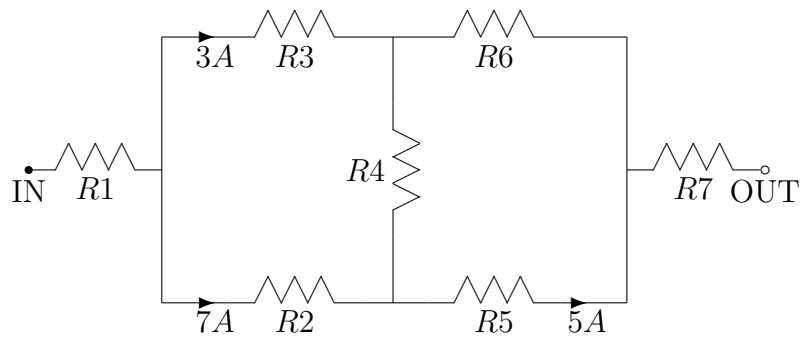


Figure 3: Kirchoff's First Law

4 Resistance in Parallel Circuits

Assume R1 and R2 are connected in parallel across a voltage source V from which a current I is drawn.

From Kirchoff's First Law,

$$I = I_1 + I_2$$

Let R_t represent the joint resistance presented by R1 and R2 in parallel.
 By Ohm's Law,

$$I = V/R_t$$

$$I_1 = V/R_1$$

$$I_2 = V/R_2$$

$$\text{So } V/R_t = V/R_1 + V/R_2$$

$$1/R_t = 1/R_1 + 1/R_2$$

$$\text{This is often written as } R_t = R_1 R_2 / (R_1 + R_2)$$

In general, $1/R_t = 1/R_1 + 1/R_2 + 1/R_3 + \dots + 1/R_n$
for any n resistors connected in parallel.

Note that the result of putting resistors in parallel is to create a total resistance that is lower in value than any of the individual resistances.

5 Power

The rate at which work is done in an electrical circuit is called power. It corresponds to the rate of energy dissipation due to resistance presented to current flowing in the circuit.

The unit of electric power is the Watt.

One Watt is the rate at which work is being done in a circuit in which a current of one ampere is flowing when an emf of one volt is applied.

The power consumed in a resistor is determined by the voltage measured across it multiplied by the current flowing through it.

$$P = VI$$

$$\text{Since } V = IR, P = I^2 R$$

$$\text{Since } I = V/R, P = V^2/R$$

Many devices have a wattage rating to indicate the rate of power dissipation they can withstand. Thus, for instance, resistors of the same resistance are available in a range of wattage ratings.

6 Alternating Current

Alternating voltage and alternating current (AC) take the form of a sinusoidal wave about a zero voltage level.

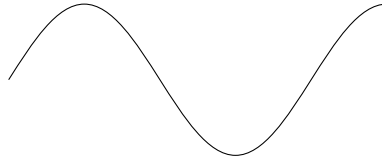


Figure 4: Sine wave

The mains electricity supply voltage alternates at a frequency of 50 cycles per second, or 50Hz (Hertz).

The effective value of alternating voltage or current, in relation to its peak value, is its root mean square value (RMS).

$$I_{rms} = I_{max}/\sqrt{2} = 0.707I_{max}$$

$$V_{rms} = V_{max}/\sqrt{2} = 0.707V_{max}$$

In AC circuits containing only resistance, current and voltage are in phase. The same laws apply as for DC circuits.

$$\text{Average power} = [V_{max}/\sqrt{2}] * [I_{max}/\sqrt{2}] = V_{max}I_{max}/2 = \text{half maximum power}$$

7 Inductors

When the current in an electric circuit changes, the circuit opposes the change. One of the circuit properties that opposes the change is called inductance.

As current flow in a conductor increases, it gives rise to an increasing magnetic field around the conductor.

The direction of the magnetic force can be seen by pointing the thumb of the right hand in the direction of current flow (from positive to negative) and wrapping the fingers around the conductor. The direction of the magnetic field follows the fingers.

When a magnetic field moves across a conductor, it induces an emf in the conductor. As the magnetic field increases, it creates a counter-emf in the conductor that opposes the emf giving rise to the current. This is self-induction, and its effect is magnified if the conductor is coiled so that its magnetic fields overlap many points on the conductor.

Inductance affects DC circuits whenever the current is switched on or off. Collapsing fields in DC circuits can generate very high induced emf.

In an inductive circuit, when current increases, energy is stored in the magnetic field.

When current decreases, the circuit gives up energy from the magnetic field.

In an inductor coil, inductance depends on the number of turns, the spacing between turns, the shape and diameter of the coil, the core material, wire size and the number of layers of windings.

7.1 Inductive Rise Time

In a resistive circuit, current rises to its maximum value almost immediately when power is switched on. In an inductive circuit (which will always contain a certain amount of resistance), there is a delay in the rise to maximum current due to self- inductance.

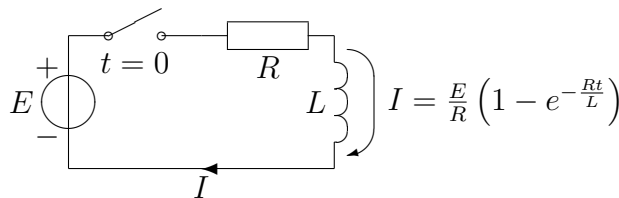


Figure 5: RL Circuit

The time for the current to rise to 63.2% ($1/e^1$) of its maximum is called the *time constant* and is given by L/R . A similar effect occurs when power is switched off. In this case, the time constant gives the time for current to fall to 36.8% of its maximum value. In these calculations, L is expressed in Henries, R in Ohms and time in seconds.

¹e, Euler's number, is irrational, approximates to 2.71828 and can be defined as the limit of $(1 + 1/n)^n$ as n approaches infinity

7.2 Inductive Reactance

Due to inductance in an AC circuit, peak current levels are lower than they would be if the circuit was only resistive in nature. This is because current has less time to rise to its maximum before it has to drop again. The higher the frequency, the more exaggerated this effect becomes. Inductive reactance is the opposition offered by an inductor to current flow. It depends not just on the inductance, but also on frequency. Inductive reactance²⁾ is given by $X_L = 2\pi fL$ Here X_L is in ohms, L in Henries, f in Hertz

Furthermore, the current waveform lags behind the voltage waveform so that the two waves are out of phase. In the case of pure inductance, current will lag voltage by 90 degrees or $\pi/2$ radians. In general, the angle by which current lags voltage is called the phase angle, denoted by Φ . The phase angle can be calculated from the formula $\tan \Phi = X_L/R$

7.3 Power in circuits containing inductors

In a circuit containing only pure inductance, current lags voltage by 90 degrees, or $\pi/2$ radians. The power wave for a circuit can be obtained by multiplying together the current and voltage values at each instant. In the case of a purely resistive circuit, the current and voltage waveforms are in phase and the power waveform is entirely positive.

In a circuit containing reactance, however, current will lag voltage and there will be some time intervals for which voltage is positive when current is negative and vice versa. During these intervals, the power waveform will be negative.

Positive power represents power being stored and/or dissipated in a circuit.

Negative power represents power being returned to the power source.

The power dissipated (used up) in a circuit can be found by subtracting the negative power from the positive power.

In a circuit containing an inductor, some of the power supplied to the circuit is used to build up a magnetic field in the inductor. When this field collapses, it returns power to the power source as negative power.

7.3.1 Power Factor

In a purely resistive circuit, current and voltage are in phase and power is always positive. The rms power is given by $P_{rms} = V_{rms}I_{rms}$, giving power

²⁾ π is irrational, approximates to 3.14159 and can be defined as the ratio of the circumference of a circle to its diameter

measured in Watts, as for DC circuits.

In a circuit with some inductance, multiplying the rms current and voltage together does not give the true power dissipation, since the waveforms are out of phase.

However, the formula $P_{rms} = V_{rms}I_{rms}$ can be used to give the Apparent Power, which is then measured in Volt-Amperes (VA) rather than watts.

The true power dissipation can be calculated as $P_{rms} = I_{rms}^2 R$ and will be less than or equal to the Apparent Power.

The ratio of True Power to Apparent Power is called the Power Factor.

In a purely resistive circuit consisting of a power supply of 12V rms, a current of 1.2A rms and a resistance of 10 ohms, the True Power is $I^2 R = 14.4W$, the Apparent Power is $P_{rms} = V_{rms}I_{rms} = 14.4 VA$ and the resulting power factor will be $P_{true}/P_{apparent} = 1$.

In a purely reactive circuit consisting of a power supply of 12V rms, a current of 1.2A rms and a reactance of 10 ohms, the True Power is $I^2 R = 0W$, the Apparent Power is $P_{rms} = V_{rms}I_{rms} = 14.4 VA$ and the resulting power factor will be $P_{true}/P_{apparent} = 0$.

8 Capacitors

Just as inductors tend to oppose changes in current flow in electric circuits, there are also elements that oppose changes in voltage. These elements are called capacitors.

Consider two parallel metal plates A and B connected in a circuit with a 5V battery (the negative terminal of which connected to ground) and two switches, SW1 and SW2, so that SW1 lies between Plate A and +5V and SW2 lies between plate B and ground.

Let both switches be initially open.

Before any switch is closed, the voltage across the battery terminals is 5V. The voltage between Plate A and ground is 0V. The voltage between plate B and ground is 0V.

If SW1 is closed at time t_1 , plate A's potential is 5V. Plate B will reach 5V after a time interval during which the charge on Plate A pushes electrons away from Plate B. Suppose Plate B reaches 5V at time t_2 .

If SW2 is then closed at t_3 , Plate B's potential changes to 0V with respect to ground over an interval terminating at time t_4 .

Current flow between VCC and Plate A can be viewed as a spike at t_1 diminishing to 0 at t_2 .

Current flow between Plate B and GND can be viewed as a spike at t_3 diminishing to 0 at t_4 .

If SW1 and SW2 are now opened, plates A and B will be isolated. A 5V charge remains across AB.

If the terminals of plates A and B are now reversed in the circuit and the switches closed again, a temporary current flow will result, in the opposite direction.

If this entire procedure is repeated at, for example, a rate of 50 times per second, then bursts of current will flow 50 times in a positive direction and 50 times in a negative direction during each second.

Thus, although the parallel plates A and B represent an open circuit to a DC source, they allow current to flow when connected across an AC source.

8.1 Factors affecting capacitance

Capacitors consist of parallel plates separated by an insulating material that is referred to as a dielectric.

- Increasing the plate area increases the capacitance
- Increasing the distance between the plates decreases the capacitance
- Changing the dielectric material changes the capacitance

When capacitors are connected in series, the total capacitance is obtained by using a formula similar to that describing resistors in parallel.

$$1/C_t = 1/C_1 + 1/C_2 + 1/C_3 + \dots + 1/C_n$$

Capacitors connected in parallel have the effect of increasing the total plate area:

$$C_t = C_1 + C_2 + C_3 + \dots + C_n$$

8.2 Time Constant

When a capacitor is connected to a voltage source, the voltage across the terminals of the capacitor does not instantly reach the value of the applied voltage. This is because every circuit contains some resistance, and this resistance slows down the charging of the capacitor plates. The larger the resistance in the circuit, the longer it takes the capacitor to charge. The time taken to charge a capacitor to 63.2% of its final value is called the time constant of the circuit and equals RC , where R is expressed in ohms and C in Farads.

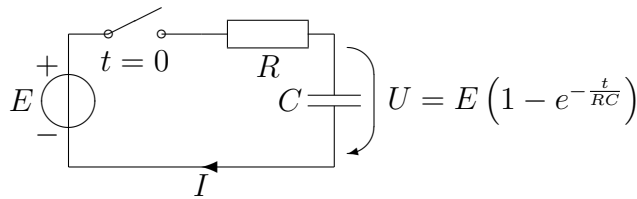


Figure 6: RC Circuit

8.3 Capacitive Reactance

A capacitor allows current to flow in an AC circuit.

- Current flow is highest when the capacitor plates begin to charge and lowest as the charge reaches its maximum.
- Switching the polarity of the voltage applied to the capacitor allows the highest value of current to flow more frequently.
- Thus, the AC current flowing through a capacitor increases with the frequency of the applied voltage. Therefore, the reactance of a capacitor is inversely proportional to frequency.
- For a fixed voltage frequency, current through a capacitor increases with increasing capacitance

Therefore, the reactance of a capacitor is inversely proportional to its capacitance. The formula for capacitive reactance is given by

$$X_C = 1/2\pi fC$$

where f is the frequency in Hertz, C is the capacitance in Farads and X_C is in Ohms.

8.4 Phase angle in a capacitive circuit

Since current has to flow in a capacitive circuit before the capacitor charges, current is said to lead the voltage in this circuit.

In a purely capacitive circuit, current will be at its peak when voltage is zero and current will be zero when voltage is at its peak.

This means that voltage lags behind current by 90 degrees ($\Pi/2$ radians) in a purely capacitive circuit.

When resistance is added to the circuit, the amount by which voltage leads current is reduced and the phase angle is given by

$$\tan \Phi = X_C/R$$

8.5 Power in a capacitive circuit

As in the case of inductive circuits, the power wave for a pure capacitive circuit can be obtained by multiplying together the current and voltage values at each instant.

The resulting power wave has the same zero axis as voltage and current, and oscillates at twice the frequency of either voltage or current.

The ratio of true power to apparent power gives the power factor. As for inductive circuits, it can be calculated as $I^2_{\text{rms}}R/V_{\text{rms}}I_{\text{rms}}$.

Clearly, in a purely capacitive circuit the power factor is 0 since the resistance is 0 and power used to charge the capacitor in the positive cycle is returned to its source in the negative cycle.

If resistance is added to the circuit, the phase angle between voltage and current decreases and some power is dissipated in the circuit. In that case, the power factor lies somewhere between 0 and 1.