Logic/Formal Language

Has 3 basic elements

- Syntax
  - well-formed formulae
- Semantics
  - the meaning of formulae
- Proof rules/’calculation’ rules
  - syntactic transformations

Logic

- A logic is an artificial language
- Syntax
  - there is a correct way of constructing a formula in this language
  - well-formed formulae are those that satisfy the syntactic rules
  - Backus-Naur Format is a useful notation for describing syntax (used c.1960 for Algol)

Syntax

- An alphabet to represent propositions
- The operators (functions)
  - e.g. $\neg, \vee, \rightarrow$
- Well-formed formulae?

$$B \wedge \neg A(B \vee C)$$

$$P \rightarrow (Q \vee R) \rightarrow D$$

$$X \wedge \neg A \wedge (B \neg \vee C) \rightarrow D$$

$$(H \vee J) \rightarrow \neg (K \vee L) \rightarrow P$$

Syntax

- A syntax given in Backus-Naur Format (BNF):

$$\begin{aligned}\varphi ::== {} & P \mid \\ & \neg\varphi \mid \\ & \varphi \wedge \varphi \mid \\ & \varphi \vee \varphi \mid \\ & \varphi \rightarrow \varphi \mid \\ & \varphi \Leftrightarrow \varphi \mid \\ & (\varphi)\end{aligned}$$

Propositional Calculus

Proposition: statement that is either true or false

The system is in idle mode.

The shutdown command has been entered.

The user is not authorised.

The validation log file is created.

No acknowledge messages have been received.

Propositions

Temperature exceeds 1000C.	*TempOver1k*
The input valve is closed.	*InValveClosed*
The reactor is shut down.	*ReactorShutDown*
The file has been created.	*FileCreated*

Logical propositions are very similar to boolean variables in a programming language.

Propositional Calculus

The file has not been created

$$\neg FileCreated$$

The input valve is open
or the output valve is closed

$$InValveOpen \vee (\neg OutValveOpen)$$

The input valve is open
and the reactor is shut down

$$InValveOpen \wedge (\neg \mathrm{Re}\, actorShutDown)$$

Whenever
the temperature exceeds 1000C
and the input valve is open
then the reactor is shut down.

$$(TempOver1k \wedge InvalveOpen) \rightarrow \\ \mathrm{Re}\, actorShutDown$$

Propositional operators/connectives (intuitive meaning/semantics)

A	
$\neg A$	negation, *not*
$A \vee B$	disjunction, *or*
$A \wedge B$	conjunction, *and*
$A \rightarrow B$	implication, *implies*
$A \Leftrightarrow B$	equivalence, *if and only if*
$A \vee_{ex} B$	*exclusive or*

Logic

- The well-formed pieces of syntax must have some meaning
- Semantics
  - assigns a meaning to syntactic elements
  - this intuitively is a “real-world” meaning … though it may be another, more “standard” part of the world of mathematics ...

Semantics

A	
$\neg A$	is true when A is false
$A \vee B$	is true when A or B is true (holds)
$A \wedge B$	is true when A and B are true
$A \rightarrow B$	is true if B is true whenever A holds
$A \Leftrightarrow B$	is true if the truth of A and B correspond

Truth Tables

A	B	$\neg A$	$A \wedge B$	$A \vee B$	$A \rightarrow B$	$A \Leftrightarrow B$
F	F	T	F	F	T	T
F	T	T	F	T	T	F
T	F	F	F	T	F	F
T	T	F	T	T	T	T

Implication

A	B	$A \to B$	
F	F	T	(i)
F	T	T	(ii)
T	F	F	(iii)
T	T	T	(iv)