Logic: an informal introduction

Ancient Greece

Greek (Athenian) democracy, c. 6th century BC

Assembly with motions for debate, discussion

The Sophists

– study of rhetoric

``*Logic differeth from rhetoric..in this, that logic handleth reason exact and in truth, and rhetoric handleth it as it is planted in popular opinions and manners.*" Francis Bacon in 1605

Logic

- General principles
  - true premises (accepted truths/facts)
  - valid argument (valid ways of combining facts)

- Foundation for the study of knowledge

Logic

- Aristotle
  - 19 syllogisms

  a syllogism is a correct form of argument
- Euclid
  - formulation of geometry

    (c.f. more modern Non-Euclidean geometry)
  - definitions, axioms, postulates
  - rigorous argument

Aristotle Syllogism Example

- Correct form of argument:

  All men are mortal

  Socrates is a man

  Therefore, Socrates is mortal

  (Darii)

Aristotle's Syllogisms – the foundation for the study of logic, virtually unchallenged for 2000 years

George Boole 1815-1864

``*The design of the following treatise is to investigate the fundamental laws of the operations of the mind by which reasoning is performed; to give expressions to them in the symbolic language of a calculus, and upon this foundation to establish the science of logic and construct its methods.''*

The Mathematical Analysis of Logic, 1847

Symbolic logic

- Attempt to formulate logic in terms of a mathematical language.
- Rules of inference were modeled after various laws for manipulating algebraic expressions:

similarity of set union and intersection and that of numerical addition and multiplication, e.g.,

a(b + c) = (ab) + (ac) is similar to

$$X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$$

Boolean Logic

- model human reasoning
- ‘calculate’ reasoning for compound statements
- correctness depends on how propositions are connected

Boolean Logic

If the electricity is operating and
the switch is down and
the light-bulb is working then the light is on

If the electricity is operating and
the switch is down and the light is not on
then the light-bulb is not working

Is this a correct argument ???

Natural Language vs. Logic

- Not all sentences are logical statements
  - Turn off your mobile phones, please.
  - Where is the remote control?
  - What a cool mobile!
- Connectives do not correspond exactly

  It was raining but we played golf.
  - It was raining *AND* we played golf.

  He didn't go to the concert or go to the pub.
  - He didn't go to the concert AND
    he didn't go to the pub

Paradoxes

- The Liar's Paradox:

  ``This sentence is a lie."

- The Sophist's Paradox.

  A Sophist lawyer sues for the return of his fees by the university that educated him. He argues that he must win, since, if he loses, the university didn't educate him well enough, and doesn't deserve the money. The university argues that he must lose, since, if he wins, he was educated well enough and therefore should pay for it.

Truth procedure?

David Hilbert, the most prominent mathematician of his time, proposed a grand program to devise a single formal procedure that would derive all mathematical truth:

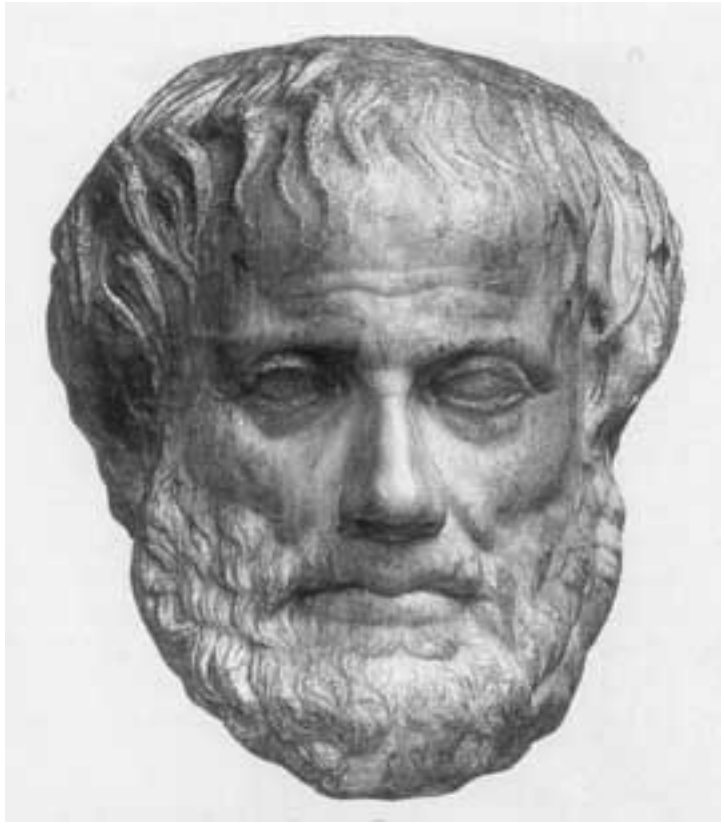
``Once a logical formalism is established one can expect that a systematic, sotosay computational, treatment of logic formulas is possible, which would somewhat correspond to the theory of equations in algebra."

Problems

- Russell's paradox
- Godel's Incompleteness Theorem

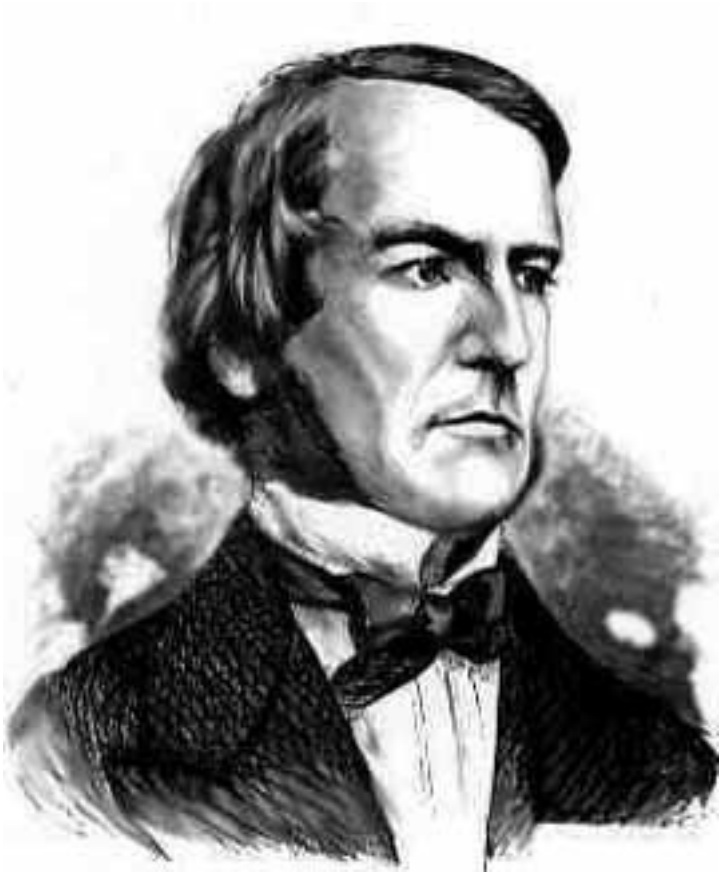
  in any formal system powerful enough to form statements about what it can prove, there will always be true statements that the system can express but can't prove.
- Church and Turing … computability

Aristotle (384 **BC**-322 **BC**)



Euclid (c.325-c.270 **BC**)

Boole



Church