## CS6323: Problem Set

## **Queueing Network Questions**

- 1. Consider the queueing network shown in Figure 1. All arrival and service rates are assumed to be exponential. A job entering the system can exit after being processed by station 1 (which has rate  $\mu$ 1) with probability p1; otherwise the job can get recycled into the system and pass through station 2 just once.
  - a. If  $\mu 1=\mu 2=20$ , and  $\lambda=10$ , and p1=1/2, compute the mean time a job will be in the system.
  - b. If we want to halve the mean time in the system, what service rate do we need for stations 1 and 2 if  $\mu$ 1= $\mu$ 2?
  - c. Can we halve the mean time in the system by changing p1?
  - d. If we allow jobs to be recycled multiple times, does this reduce the quality of service?
  - e. If  $\mu 1=\mu 2=20$ , and  $\lambda=10$ , and p1=1/2, and we reduce the buffer size of station1 to 2, and jobs arriving from outside get dropped if there is no buffer space, what is the mean number of dropped jobs?

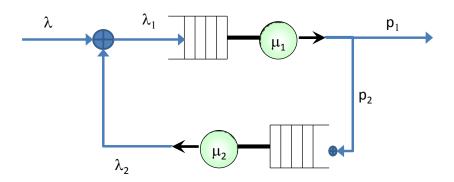


Figure 1: simple network

- 2. Consider a cloud computing system S that consists of 2 data centers, each with 1000CPUs. We say that S is functioning of at least one data center is functional. S can earn revenue of €100M/hour if both data centers are working, €50M/hour if one data center is working, but must pay a penalty of €100M/hour if both data centers are down.
  - a. If we have an arrival rate of  $\lambda$ =100k jobs per hour, and each data center has a service rate of  $\mu$ =75K jobs per hour, compute the expected revenue per hour for S.
  - b. If we lose our job buffer so that any job is lost if both data centers are busy, compute the expected revenue per hour.
- 3. Assume that a service station has an outside waiting area where cars queue before they get petrol, at one of two stations. All arrival and service rates are assumed to be exponential. Each petrol pump has a waiting area as well. The queueing model for this system is shown in Figure 2. Only 1 car can be waiting for service at a petrol pump, in addition to the car pumping petrol.

- a. If  $\mu 1=\mu 2=20$ , and  $\lambda=20$ , and p1=1/2, compute the mean time a job will be in the system.
- b. If  $\lambda$  increases to 30, compute the mean time a job will be in the system and the mean number in the station.
- c. The station owner forces all cars not pumping petrol to wait at the outside waiting area, decreasing the buffer for each pumping station to 0. Compute the mean number in the outside waiting area if  $\lambda$ =30.
- d. Compute the distribution over the number of cars in the system.
- e. If a driver finds 2 or more cars waiting at the outside waiting area they will find another petrol station. Compute the mean number of lost customers.
- f. The station owner changes one pump to diesel, with rate  $\mu 2$ =30. Compute the distribution over the number of cars in the system for the two pump types.

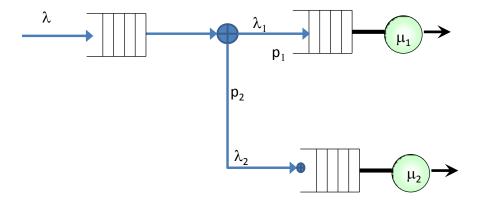


Figure 2: queueing system for petrol station