CS2205: Theory of Computation
Problem Class IX

1. Determine the weakest precondition P that satisfies the following. Simplify your answers as much as you can.
   (a) $(\!| P |\!)$ **if** $i = 1$ $j := i$ **else** $j := i + 1$ $(\!| j = 1 |\!)$
   (b) $(\!| P |\!)$ **if** $i = m$ $m := k$ **else** $j := k$ $(\!| k = j \wedge j = m |\!)$
   (c) $(\!| P |\!)$ **if** $i > 1$ $i := i + 1$ **else** $j := j + i$ $(\!| j \leq i + 1 |\!)$
2. Prove that
   (a) $\vdash_{\text{par}} (\!| i \neq 0 |\!)$ *ProgA* $(\!| j = 1 |\!)$ where *ProgA* is as follows:

```
if i = 0
{    j := 0
}
else
{    j := 1
}
```

   (b) $\vdash_{\text{par}} (\!| x \geq 0 \wedge y \geq 0 |\!)$ *ProgB* $(\!| x \geq 0 \wedge y \geq 0 |\!)$ where *ProgB* is as follows:

```
if x > y
{    x := x - y
}
else
{    y := y - x
}
```


3. Prove that $\vdash_{\text{par}} (\!| x \geq 0 |\!)$ *ProgC* $(\!| s = 2^x |\!)$ where *ProgC* is below. In other words, prove that *ProgC* computes 2^x (for non-negative x). Use $s = 2^i$ as the loop invariant.

```
s := 1
i := 0
while i ≠ x
{    s := s + s
     i := i + 1
}
```

4. Prove that $\vdash_{\text{par}} (\!| n \geq 0 |\!)$ *ProgD* $(\!| s = n^2 |\!)$ where *ProgD* is below. In other words, prove that *ProgD* computes n^2 (for non-negative n). Use $s = i^2$ as the loop invariant.

```
i := 0
s := 0
while i ≠ n
{    i := i + 1
     s := s + 2i - 1
}
```