CS2205: Theory of Computation
Problem Class VIII

1. In this exercise, I give you an (informal) Problem Specification then I give you a corresponding (formal) Program Specification as a Hoare triple. First, I want you to give me a trivial MOCCA program which satisfies the Program Specification, but which is probably not what the person who wrote the Problem Specification intended. Then, I want you to rewrite the Program Specification so that it more accurately captures the intention behind the Problem Specification.

   Huh?

   Here's an example. Here's a Problem Specification:

   Problem VIII.1.
   Parameters: *An integer x.*
   Returns: x^2

   Here's a Program Specification:

   $$(\!|\,\mathbf{True}\,|\!)\; C\; (\!|\, z = x^2 \,|\!)$$

   (a) The first thing you would give is a trivial MOCCA program that satisfies the Program Specification, e.g.:

   ```
   x := 3;
   z := 9;
   ```

   (b) Then you give a Program Specification that more accurately captures the intention behind the Problem Specification, e.g.:

   $$(\!|\, x = x_0 \,|\!)\; C\; (\!|\, z = x_0^2 \,|\!)$$

   Now it's your turn.

   (a)

   Problem VIII.2.
   Parameters: *Two integers, x and y.*
   Returns: $x + y$

   Program Specification:

   $$(\!|\,\mathbf{True}\,|\!)\; C\; (\!|\, z = x + y \,|\!)$$

   (b)

   Problem VIII.3.
   Parameters: *Two integers x and y, where $y \geq 0$.*
   Returns: x^y

   Program Specification:

   $$(\!|\, y \geq 0 \,|\!)\; C\; (\!|\, z = x^y \,|\!)$$

2. Determine the weakest precondition P that satisfies
   (a) $(\!|\, P \,|\!)\; i := i + 1\; (\!|\, i > 0 \,|\!)$
   (b) $(\!|\, P \,|\!)\; z := x\; (\!|\, z \geq x \wedge z \geq y \,|\!)$
   (c) $(\!|\, P \,|\!)\; i := (lo + hi) \operatorname{div} 2\; (\!|\, lo \leq i \wedge i \leq hi \,|\!)$
   (d) $(\!|\, P \,|\!)\; y := x\; (\!|\, x = y \,|\!)$
   (e) $(\!|\, P \,|\!)\; x := y\; (\!|\, z = y \,|\!)$

3. Determine the weakest precondition P that satisfies
   (a) $(\!|\, P \,|\!)\; x := x + 1; y := y + 1\; (\!|\, x = y \,|\!)$
   (b) $(\!|\, P \,|\!)\; i := i + 1; j := j - 1\; (\!|\, i + j = 0 \,|\!)$
   (c) $(\!|\, P \,|\!)\; i := i + 1; j := j - 1\; (\!|\, i \times j = 0 \,|\!)$
   (d) $(\!|\, P \,|\!)\; x := 2x + 1; y := y - 1\; (\!|\, y = 3x + 1 \,|\!)$
   (e) $(\!|\, P \,|\!)\; i := i - 1; z := z \times j\; (\!|\, z \times j^i = c \,|\!)$

4. Prove that
   (a) $\vdash_{\text{par}} (\!|\,\mathbf{True}\,|\!)\; i := 0\; (\!|\, i = 0 \,|\!)$
   (b) $\vdash_{\text{par}} (\!|\,\mathbf{False}\,|\!)\; i := 1\; (\!|\, i = 0 \,|\!)$
   (c) $\vdash_{\text{par}} (\!|\, i < n \,|\!)\; i := i + 1\; (\!|\, i \leq n \,|\!)$

5. Prove that
   (a) $\vdash_{\text{par}} (\!|\,\mathbf{True}\,|\!)\; \mathit{ProgA}\; (\!|\, y = 3x \,|\!)$ where *ProgA* is as follows:

   ```
   y := x;
   y := x + x + y;
   ```

   (b) $\vdash_{\text{par}} (\!|\, xy + pq = n \,|\!)\; \mathit{ProgB}\; (\!|\, xy + pq = n \,|\!)$ where *ProgB* is as follows:

   ```
   x := x - p;
   q := q + y;
   ```

(c) $\vdash_{\text{par}} (\!| x > 1 |\!)\ \mathit{ProgC}\ (\!| y > 0 \wedge x > y |\!)$ where ProgC is as follows:

```
a := 1;
y := x;
y := y − a;
```

6. Which of these holds for all P, C and Q?

(a) $\models_{\text{par}} (\!| \mathbf{True} |\!)\, C\, (\!| Q |\!)$
(b) $\models_{\text{tot}} (\!| \mathbf{True} |\!)\, C\, (\!| Q |\!)$
(c) $\models_{\text{par}} (\!| P |\!)\, C\, (\!| \mathbf{True} |\!)$
(d) $\models_{\text{tot}} (\!| P |\!)\, C\, (\!| \mathbf{True} |\!)$
(e) $\models_{\text{par}} (\!| \mathbf{False} |\!)\, C\, (\!| Q |\!)$
(f) $\models_{\text{tot}} (\!| \mathbf{False} |\!)\, C\, (\!| Q |\!)$
(g) $\models_{\text{par}} (\!| P |\!)\, C\, (\!| \mathbf{False} |\!)$
(h) $\models_{\text{tot}} (\!| P |\!)\, C\, (\!| \mathbf{False} |\!)$