CS2205: Theory of Computation
Problem Class VII

1. Show that the following derivations hold using the rules of the natural deduction system covered in lectures.

   Hints.

   - For exercises (a)–(d), you will need only the rules $\wedge$-I, $\wedge$-E-L, $\wedge$-E-R, $\Rightarrow$-E, $\vee$-I-R, $\vee$-I-L, $\Leftrightarrow$-E=L, $\Leftrightarrow$-E-R and $\neg$-E.
   - For exercises (e)–(f), you will *additionally* need $\Rightarrow$-I and you may also choose to use REPETITION.
   - For exercises (g)–(h), which are not easy, you will *additionally* need $\neg$-I and then also $\vee$-E. For these two, ask for further hints if you get stuck.

   (a) $\{p_1 \wedge p_2, p_3 \wedge p_4\} \vdash p_1 \wedge p_4$

   (b) $\{p, p \Rightarrow q, q \Leftrightarrow r\} \vdash q \wedge r$

   (c) $\{\neg\neg q\} \vdash q \vee r$

   (d) $\{p_1 \wedge p_2, p_1 \Rightarrow p_3, p_2 \Rightarrow p_4\} \vdash p_3 \wedge p_4$

   (e) $\{(p \wedge q) \Rightarrow r, p\} \vdash q \Rightarrow r$

   (f) $\{p\} \vdash q \Rightarrow p$

   (g) $\{p_1 \Rightarrow p_2, \neg p_3 \Rightarrow \neg p_2, p_3 \Rightarrow \neg p_4\} \vdash p_4 \Rightarrow \neg p_1$

   (h) $\{p \vee q, \neg q\} \vdash p$

2. Prove the following.

   (a) $\vdash r \Rightarrow (p \Rightarrow (q \Rightarrow r))$

   (b) $\vdash (q \Rightarrow r) \Rightarrow ((p \Rightarrow q) \Rightarrow (p \Rightarrow r))$

   (c) $\vdash p \vee \neg p$

3. Show whether this inference rule is sound or not:

$$\frac{(W_1 \wedge W_2) \Rightarrow W_3}{W_1 \Rightarrow W_3}$$