CS2205: Theory of Computation
Problem Class VI

1. Consider the following DECAFF algorithm which uses a Boolean variable, *equal*, to determine whether two arrays of integers, a and b, are equal to each other, i.e. whether they have exactly the same contents in exactly the same order.

Algorithm: EQUALARRAYS(a, b)

Parameters: An array of integers $a[0 \ldots m]$ and an array of integers $b[0 \ldots n]$, where $m \geq 0, n \geq 0$.
Returns: true if the two arrays have exactly the same contents in exactly the same order; **false** otherwise.

```
{   equal := (m = n);
    if equal
    {   for i := 0 upto m
        {   equal := (a[i] = b[i]);
        }
    }
    return equal;
}
```

Is the program correct? If you think it is not, give an example instance on which it returns the wrong answer.

2. Say which of the following expressions are *syntactically well-formed formulae* of propositional logic and which are not.

   (a) $(p_1 \wedge p_2 \vee \neg p_2)$
   (b) $(\neg p_1 \wedge p_2)$
   (c) $(p_1 \Rightarrow p_2 \vee p_3)$
   (d) $p_1 \vee p_2 \Rightarrow p_3 \wedge p_4$
   (e) $p_1 \Rightarrow p_2 \Rightarrow p_3$

3. For those expressions from above that you said were well-formed, rewrite the expressions, inserting additional parentheses to make explicit the way that the operator precedence and associativity conventions given in your notes would affect the evaluation order.

4. In this question,

$$\begin{aligned} p_1 &=_{\text{def}} \text{Clyde is going to the beach} \\ p_2 &=_{\text{def}} \text{Clyde is going to the cinema} \\ q_1 &=_{\text{def}} \text{Flopsy is going to the beach} \\ q_2 &=_{\text{def}} \text{Flopsy is going to the cinema} \end{aligned}$$

Write each of the following English statements in symbolic form, i.e. as wffs of propositional logic.

In those cases where you believe that the full meaning is not captured, give the best translation into symbolic form that you can, but add a note in English to your answer that records what aspect of the meaning you think is not being adequately captured.

   (a) Clyde is going to the beach but Flopsy is going to the cinema.
   (b) Clyde is going to the beach or the cinema, and so is Flopsy.
   (c) If Clyde goes to the beach while Flopsy does not, Flopsy will go to the cinema.
   (d) Clyde is not only going to the beach but to the cinema too.
   (e) If Clyde goes to the beach if Flopsy goes to the beach, then Clyde is going to the beach.
   (f) Clyde goes to the beach if Flopsy does, and the other way round.
   (g) Clyde is going to the beach if Flopsy doesn't.
   (h) Clyde goes to the beach only if Flopsy doesn't.
   (i) Clyde goes to the beach exactly if Flopsy doesn't.
   (j) Clyde goes to the beach unless Flopsy does.

5. Draw truth tables for the following wffs *and hence* state whether each is valid or invalid, and satisfiable or unsatisfiable.

   (a) $p \vee (p \Rightarrow q)$
   (b) $p \wedge \neg(p \wedge p)$
   (c) $\neg(\neg(p \wedge q) \wedge \neg(p \vee q))$
   (d) $((p \vee \neg q) \vee (\neg p \vee q)) \Rightarrow r$

6. Use truth tables, *plus a sentence or two of explanation*, to show:

   (a) $\{p \Leftrightarrow q, q\} \models p$
   (b) $\{p \Rightarrow \neg q, \neg r \Rightarrow p, q\} \models r$
   (c) $\{p \Leftrightarrow \neg q\} \not\models p \Rightarrow q$
   (d) $\{(q \vee p) \Rightarrow r\} \models q \Rightarrow r$

7. *Without drawing truth tables*, state whether the following hold. Include some words of explanation in your answer.

   (a) $\{(p \vee q) \Rightarrow (q \wedge \neg r)\} \models (p \vee q) \Rightarrow (q \wedge \neg r)$

   (b) $\{\neg((p \vee q) \Rightarrow (q \wedge \neg r))\} \models (p \vee q) \Rightarrow (q \wedge \neg r)$

   (c) $\{(p \vee q) \Rightarrow (q \wedge \neg r), p \vee r\} \models p \vee \neg p$

   (d) $\{p, \neg p\} \models (p \vee q) \Rightarrow (q \wedge \neg r)$

8. Use truth-tables to show that the following equivalences hold. Include enough columns to show your working.

   (a) $(W_1 \wedge W_2) \Rightarrow W_3 \equiv W_1 \Rightarrow (W_2 \Rightarrow W_3)$

   (b) $W_1 \equiv (W_1 \wedge W_2) \vee (W_1 \wedge \neg W_2)$

   (c) $W_1 \Leftrightarrow W_2 \equiv \neg W_1 \Leftrightarrow \neg W_2$

9. Repeat the above. This time show that the equivalences hold algebraically using the laws given in your notes.