CS2205: Theory of Computation
Problem Class XX (Extra Exercises)

1. We desire a Turing machine where $\Sigma = \{a, b, Y, N, \llcorner\!\lrcorner\}$ which, when started on the rightmost non-blank of a string of consecutive a's and b's, will check whether the string contains alternating a's and b's. It should not erase the original input string; it should write Y at the leftmost end of the input string if it does contain alternating a's and b's; it should write N at the leftmost end otherwise. For example, $ababa$ becomes $Yababa$, whereas abb becomes $Nabb$.

   Give Q and δ.

2. We desire a Turing machine for which $\Sigma = \{0, 1, *, \llcorner\!\lrcorner\}$. Initially, its tape will contain two equal-length binary numbers, separated by a single blank cell, and the Turing machine's head will be positioned at the rightmost non-blank. The machine will erase the two binary numbers and write the result of ANDing them onto the tape. For example, $0101\llcorner\!\lrcorner1100$ becomes 0100.

   Construct your machine from the basic symbol-writing and head-moving Turing machines and present your machine using the graphical notation.

   Hints. One idea is to overwrite one of the numbers with the answer. Another idea is to use the extra symbol $*$ to find your way back to cells you visited previously.