CS2205: Theory of Computation
Problem Class XIX (Revision)

1. A company is using an algorithm whose worst-case time complexity is $O(n^2)$. Dr. Toc offers the company an algorithm whose worst-case time complexity is $O(n \log n)$. The good doctor is furious when the company's Technical Director says, "We won't be using your new algorithm. It's slower than the one we're presently using." Can the Technical Director be right? Explain your answer in detail.

2. Decide whether or not the following hold. In each case, prove what you decide.
   (a) $2^{\log n} + \log n^3$ is $O(n)$.
   (b) $2^{n+1} + 2^n + 2$ is $O(2^{n^2})$.
   (c) 2^{n^2} is $O(2^n)$
   (d) $\log n$ is $O(\frac{1}{n})$

3. Give a big-Oh characterisation, in terms of n, of the worst-case time complexity of the following algorithms. In each case, justify your answers.

Algorithm: A(n)

```
s := 0;
c := n;
for i := 1 upto c
{   s := s + i × c;
}
c := 10;
for j := 1 upto n^2
{   for k := 1 upto c^2
    {   s := s + j × k;
        s := s div 2;
    }
}
```


Algorithm: B(n)

```
s := 0;
for i := 1 upto n div 2
{   s := s + i × 3;
    s := s × 2;
}
for j := 1 upto n
{   s := s + 1;
}
for k := 1 upto j
{   for m := 1 upto k
    {   s := s + k × m;
    }
}
```

4. For each of the algorithms in the previous question, give a formula, $t(n)$, in terms of n, that defines the worst-case time complexity of the algorithm, counting only assignments to s. There is no need to justify your answers; simply give the formulae.

5. Consider the following problem:

Problem XIX.1. *SubsetSum*
Parameters: *A finite set S of positive integers, and a positive integer, t.*
Returns: *YES if there is a subset S' of S ($S' \subseteq S$) whose members sum to t; NO otherwise.*

Assume there are n integers in S. Prove that *SubsetSum* $\in$ **NP**.

6. Dr. Toc has shown that a decision problem P_1 is polynomial-time reducible to an **NP**-complete problem P_2. The good doctor has also come up with a program that solves P_1 in polynomial-time. Has the good doctor proved that $\mathbf{P} = \mathbf{NP}$? Explain your answer.