CS2205: Theory of Computation
Problem Class XVIII

1. Consider a Turing machine where $\Sigma = \{a, \sqcup\}$, $Q = \{q_0, q_1, q_2, h\}$ and δ is:

δ	a	$\sqcup$
q_0	L, q_1	$\sqcup, h$
q_1	$\sqcup, q_2$	$\sqcup, h$
q_2	—	L, q_0

   (a) Trace the computation of this machine starting from configuration $\langle \sqcup aa, a, \sqcup, q_0\rangle$.

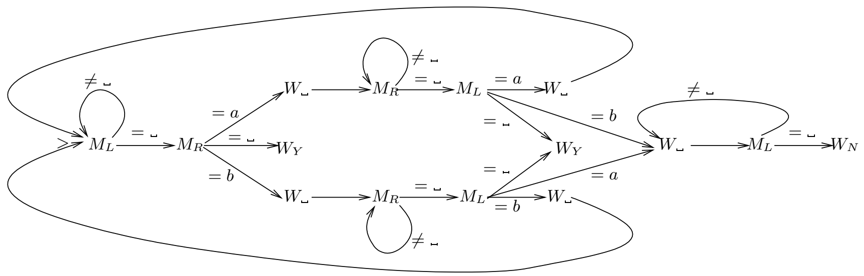
   (b) Describe informally what the machine does when started in q_0 on the rightmost non-blank of a string of consecutive a's.

2. We desire a Turing machine where $\Sigma = \{a, b, Y, N, \sqcup\}$ which, when started on the rightmost non-blank of a string of consecutive a's and b's, will:

   - erase the string; and
   - output Y if the string contained at least one a or output N otherwise; and then
   - halt.

   Give Q and δ.

3. The following Turing machine has $\Sigma = \{a, b, \sqcup\}$. Describe informally what this machine does when started on the rightmost non-blank of a string of consecutive a's and b's.



4. We desire a Turing machine where $\Sigma = \{a, b, \sqcup\}$ which, when started on the rightmost non-blank of a string of consecutive a's and b's, will write the reverse of that string onto the tape, to the right-hand end of the original string, so that the two are separated by a blank. For example, if the tape originally contains abb then, when the machine halts, the tape will contain $abb\sqcup bba$.

   Construct your machine from the basic symbol-writing and head-moving Turing machines and present your machine using the graphical notation.

5. Dr. Toc has been reading the CS2205 handouts. "Excellent handouts!" the good doctor says. "But I notice that, by your definition of Turing machines, at each step the machine can either

   - write to tape and change state, or
   - move its head and change state.

   I think you should *also* allow it to

   - neither write nor move, but just change state."

   Does it matter that we do not also allow this possibility? Explain your answer in detail.

6. Dr. Toc writes a paper in which Turing machines can, in a single step, both write and then move the head. So, the cells of the transition tables now contain three items: the symbol to be written, the direction in which to move the head, and the new state. Here's an example of one of the good doctor's machines. In this machine, $\Sigma = \{a, b, \sqcup\}$, $Q = \{q_0, h\}$ and δ is as follows:

δ	a	b	$\sqcup$
q_0	b, L, q_0	a, L, q_0	$\sqcup, L, h$

   This machine scans left, replacing a's by b's and b's by a's, until it reaches a blank.

   Explain in as much detail as you can how, *in general*, to convert the good doctor's machines into the standard Turing machines covered in lectures.