Fuzzy Control

1 An Overview of Fuzzy Control

We've had an introduction to fuzzy sets and fuzzy logic. We now see how to use fuzzy logic for reasoning within a control application. We'll exemplify with the following set of rules for controlling the throttle in a steam turbine, based on temperature and pressure readings. We use some abbreviations: T for temperature, P for pressure and adjustTh for the action of adjusting the throttle.

if T is cold $\wedge$ P is weak **then** adjustTh a medium positive amount
if T is cold $\wedge$ P is low **then** adjustTh a small positive amount
if T is cold $\wedge$ P is OK **then** adjustTh a null amount
if T is cold $\wedge$ P is strong **then** adjustTh a small negative amount
etc.

There is vagueness in both the conditions and the actions.

Fuzzy control comprises the following steps:

Sense The agent will sense its environment as usual. It's important to remember that the sensors will deliver precise numeric values (amounts of light, temperature, etc.).

Fuzzify The rule conditions make use of vague terms, not precise measurements. So the incoming values must be fuzzified (e.g. so that we can know how true it is to say that the measured temperature is a 'warm' one).

Evaluate For *every* rule, the agent then works out how true the condition is.

Activate For *every* rule, the truth of the action is determined (as a fuzzy set).

Aggregate The agent must then combine the judgements of all the rules.

Defuzzify It's important to remember that the signal that the agent will send to the device being controlled (e.g. a throttle, a brake, etc.) will be a precise numeric value (that's how devices work!). For example, the agent can't instruct a motor to turn an axle 'a lot'. Instead, it might turn the motor on for some number of seconds. So the combined judgement of all the rules must, at this stage, be defuzzified into a precise value.

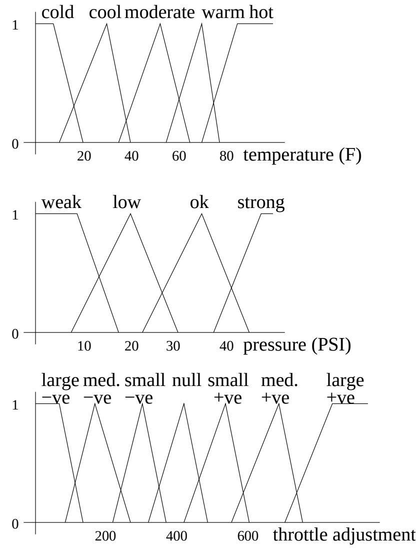
Act The agent sends the precise signal to the device that is to carry out the action.

Note from the above a major difference between fuzzy controllers and production systems. In the production systems that we were looking at before, only one rule was selected (e.g. the *first* matching rule) and its action was executed. In fuzzy control, this is generally not appropriate. Several rules may apply with different degrees of truth. We must find out *all* the rules whose conditions have some degree of truth and aggregate (combine) their judgements.

Work in fuzzy control has defined numerous ways of carrying out the evaluation, activation, aggregation and defuzzification steps. In the example that follows, we look at only one of these ways.

2 Fuzzy Control Example

We'll use the steam turbine rules from earlier. The membership functions for the fuzzy sets are plotted below:



Sense. Suppose the agent *senses* the world and finds that the temperature is $15°F$ and the pressure is 26 psi.

Fuzzify. The sensors returns precise values. But they can be *fuzzified*. From the graphs, we can see that, e.g.:

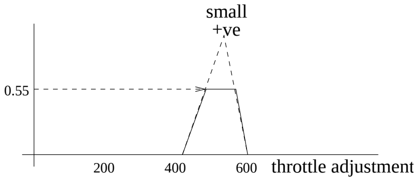
$$\begin{array}{lcllcl}
\mathcal{I}(\text{T is cold}) & = & 0.58 & \mathcal{I}(\text{P is weak}) & = & 0.0 \\
\mathcal{I}(\text{T is cool}) & = & 0.24 & \mathcal{I}(\text{P is low}) & = & 0.55 \\
\mathcal{I}(\text{T is moderate}) & = & 0.0 & \mathcal{I}(\text{P is ok}) & = & 0.28 \\
& \vdots & & & \vdots &
\end{array}$$

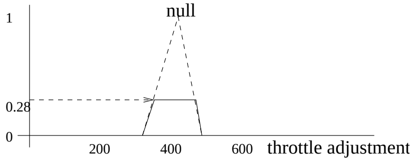
Evaluate. Now, for each rule, we *evaluate* the evidence: we work out the truth of the rule's condition. In the example, the conditions all use conjunction ($\wedge$), so the evidence is combined using $\min$.

$$\begin{array}{lcl}
\mathcal{I}(\text{T is cold} \wedge \text{P is weak}) & = & 0.0 \\
\mathcal{I}(\text{T is cold} \wedge \text{P is low}) & = & 0.55 \\
\mathcal{I}(\text{T is cold} \wedge \text{P is OK}) & = & 0.28 \\
\mathcal{I}(\text{T is cold} \wedge \text{P is strong}) & = & 0.0
\end{array}$$

Activate. The *activation* step is applied to the second and third rules, becuase they are the ones whose conditions have non-zero degrees of truth. Here, we have to work out the truth of the statement in the action of the rule, given the truth of the rule's condition. This is done by *clipping* the fuzzy set that is used in the rule's action. Clipping is best described diagrammatically.

In the case of the second rule, the truth of the condition is 0.55 (computed above), and this is used to clip the small positive throttle adjustment set. In the case of the third rule, the truth of the condition is 0.28, and this is used to clip the null throttle adjustment set:

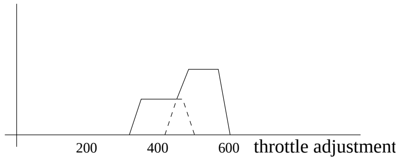




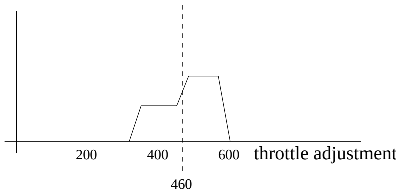
This may look like an arbitrary way of doing things, but, in fact, it has a mathematical justification.

One way of thinking about clipping is that it is an approximation of what we would get if we scaled the graph of the fuzzy set by the truth-value of the rule's antecedent.

Aggregate. The next step is *aggregation*. We must combine the judgements of all the rules that have fired (in our case, two rules). Since rules are implicitly disjoined ($\vee$), aggregation is commonly done using union (maximum). The result of this is diagrammed here:



Defuzzify. We must now determine a precise throttle adjustment value. This is the process of *defuzzification*. There are many approaches but a common one is the *centroid method*. The centroid will be the throttle adjustment value at which a vertical line would pass through the centre of the set (equal areas to left and right of the line):



Act. Finally, the agent is ready to *act*. It sends a signal to its throttle requesting an adjustment of 460 (as computed by the centroid method). And now the cycle begins again.

3 Applications

Fuzzy set theory was proposed by Lotfi Zadeh in the 1960s. But it was not until the late 1980s that fuzzy control began to look promising following a number of impressive demonstrations of its use. For example, a classic control problem is to move a vehicle back and forth in an effort to keep an inverted pendulum upright. This is usually solved by a complex set of differential equations; a simple fuzzy controller was shown to be capable of doing it.

From then on, consumer goods (especially Japanese ones) began to make much use of fuzzy control. Matsushita vacuum cleaners can adjust their suction based on dust sensor readings; Hitashi washing machines can choose wash cycle parameters (amounts of power, water and detergent) based on readings for load-weight, fabric-mix and dirtiness; Canon cameras can control lens position based on image clarity readings to give an auto-focusing facility.

4 Controversy

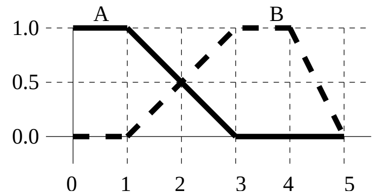
Fuzzy logic is widely accepted and used for fuzzy control. But it sees less use in other areas of AI. In particular, it is rarely used in applications where chains of reasoning are needed, i.e. where the action of one rule makes another rule fire.

People have speculated that the success of fuzzy controllers has little to do with fuzzy logic. The success, they claim, derives from the small changes made in rapid succession during repeated sense/plan/act cycles; the use of simple (non-chaining) rules that associate stimuli with responses; and the careful tuning (or learning) process that is used to 'get the numbers right' so that we get the right behaviour from the system. Fuzzy logic itself, these people claim, is incidental; a different formalism could be used instead.

Needless to say, proponents of fuzzy logic hotly contest these assertions.

Exercises

1. Suppose U, the universe of discourse, is $\{0, \ldots, 5\}$. The graph below shows the membership function for the fuzzy sets A and B:



Draw separate graphs that show the membership functions for the following fuzzy sets:

(a) A'
(b) B'
(c) $A \cup B$
(d) $A \cap B$
(e) $(A \cup B)'$
(f) $(A \cap B)'$

2. We have three statements of fuzzy logic, p, q and r. Here are their degrees of truth:

$$\mathcal{I}(p) = 0.2;\ \mathcal{I}(q) = 0.5;\ \mathcal{I}(r) = 0.7$$

Compute the degrees of truth of the following:

(a) $p \wedge q$

(b) $\neg p \vee \neg q$

(c) $\neg(p \wedge q) \vee r$

(d) $p \Rightarrow q$ (Of the 72 definitions, use the one I gave above!)

(e) $\neg p \vee q$ (Note how the answer to this one is different from the answer to the previous one, which shows that in fuzzy logic $W_1 \Rightarrow W_2$ is not always equal to $\neg W_1 \vee W_2$.)

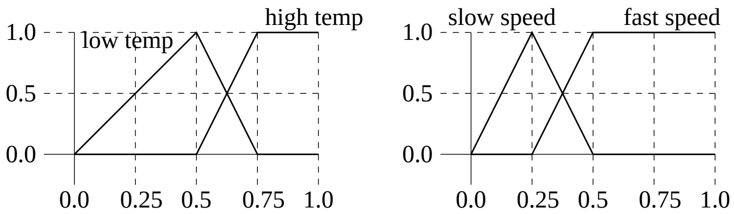
(f) $(p \Rightarrow q) \Rightarrow r$

3. We have two fuzzy rules:

if temperature is low **then** speed is slow

if temperature is high **then** speed is fast

And we have four fuzzy sets: low temperatures, high temperatures, slow speeds and fast speeds:



Suppose the temperature is 0.625. What should the speed be? (Explain your answer, including diagrams if you wish. Make an intelligent guess at where the centroid is!)