Worksheet 9

(Some) Answers

Exercise 9.1 *The validity of instances of logical axiom schemata*

1) $p \to (q \to p)$

> *Negate:* $\neg(p \to (q \to p))$
> *CFL:* $p,\ q,\ \neg p$

2) $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$

> *Negate:* $neg((p \to (q \to r)) \to ((p \to q) \to (p \to r)))$
> *CFL:* $\neg p \vee \neg q \vee r,\ \neg p \vee q,\ p,\ \neg r$

3) $(\forall X\,(p(X) \rightarrow q(X))) \rightarrow ((\forall X\,p(X)) \rightarrow (\forall X\,q(X)))$

Negate: $\neg((\forall X\,(p(X) \rightarrow q(X))) \rightarrow ((\forall X\,p(X)) \rightarrow (\forall X\,q(X))))$
CFL: $\neg p(X_1) \vee q(X_1)$, $p(X_2)$, $\neg q(a)$

□ Notes: There are often alternative (equivalent and hence equally correct) translations into FOPC to those given below. So the answers below are not "gospel". Furthermore, since I might have got the answers wrong, this is another reason not to treat the following as gospel. Come to see me if you're having problems.

Exercise 9.2 *Propositional Logic representation:*

1. $(M_g O \wedge H_2) \rightarrow (M_g \wedge H_2 O)$
2. $(C \wedge O_2) \rightarrow CO_2$
3. $(CO_2 \wedge H_2 O) \rightarrow H_2 CO_3$
4. $M_g O$
5. H_2
6. O_2
7. C
8. To show $H_2 CO_3$.

Exercise 9.3 *FOPC representation:*

1. $\forall X\,(r(X) \rightarrow l(X))$
2. $\forall X\,(d(X) \rightarrow \neg l(X))$
3. $\exists X\,(d(X) \wedge i(X))$
4. To show: $\exists X\,(i(X) \wedge \neg r(X))$.

Exercise 9.4 *FOPC representation:*

1. $\forall X\,(j(X) \rightarrow mta(X))$
2. $\forall X\,(aop(X) \rightarrow \neg j(X))$
3. To show $\forall X\,(aop(X) \rightarrow \neg mta(X))$.

Exercise 9.5 *FOPC representation:*

1. $\forall X\ ((r(X) \wedge \exists Y\ (lp(Y) \wedge s(X,Y))) \rightarrow i(X))$
2. $\forall X\ ((j(X) \wedge r(X)) \rightarrow \forall Y\ (lp(Y) \rightarrow s(X,Y)))$
3. $\exists X\ lp(X)$
4. $j(a) \wedge r(a)$
5. *To show* $\exists X\ i(X)$.

Exercise 9.6 *FOPC representation:*

1. $\forall X\ (t(X) \rightarrow \exists Y\ (p(Y) \wedge o(Y,X)))$
2. $\neg\forall X\ (n(X) \rightarrow \exists Y\ (p(Y) \wedge o(Y,X)))$
3. $\forall X\ (b(X) \rightarrow n(X))$
4. $\forall X\ (t(X) \rightarrow n(X))$
5. $\forall X\ (n(X) \rightarrow ((b(X) \vee t(X)) \wedge \neg(b(X) \wedge t(X))))$
6. *To show* $\exists X (b(X) \wedge \forall Y (p(Y) \rightarrow \neg o(Y,X)))$.

Exercise 9.7 *FOPC representation:*

1. *Originally in computer* $\neg paid(smf)$
2. *Added:* $paid(smf)$
3. *To show* $spy(pm)$.

Exercise 9.8 *Whodunnit*

Victor has been murdered, and Arthur, Bertram and Carleton are suspects. [Since only one is guilty, we get: 1. $(i(a) \vee i(b)) \wedge (i(a) \vee i(c)) \wedge (i(b) \vee i(c))$*]. [The things Arthur says will be dependent on* $i(a)$*.] Arthur says he did not do it. [i.e.* $i(a) \rightarrow i(a)$*: we can ignore this.] He says that Bertram was the victim's friend [i.e. 2.* $i(a) \rightarrow friend(b, v)$*] but that Carleton hated the victim [i.e. 3.* $i(a) \rightarrow \neg friend(c, v)$*]. Bertram says he was out of town the day of the murder, [i.e. 4.* $i(b) \rightarrow \neg in\text{-}town(b)$*] and besides he didn't even know the guy [i.e. 5.* $i(b) \rightarrow \neg know(b, v)$*]. Carleton says he is innocent [i.e.* $i(c) \rightarrow i(c)$*, which we can ignore] and he saw Arthur and Bertram with the victim just before the murder [i.e. 6.* $i(c) \rightarrow saw\text{-}with(a, v)$ *and 7.* $i(c) \rightarrow saw\text{-}with(b, v)$*]. Assuming that everyone (except possibly for the murderer) is telling the truth, who committed the crime? [i.e. 8. the goal is* $\exists x \neg i(X)$*].*

Other world knowledge is that anyone who was with Victor must have been in town: 9. $\forall X (saw\text{-}with(X, v) \rightarrow in\text{-}town(X))$*. And that friends know each other: 10.* $\forall X \forall Y friend(X,Y) \rightarrow know(X,Y)$*.*