Worksheet 6

Answers

Exercise 6.1 *Honest car dealers*

1. *There is at least one used-car dealer.*
2. *There is at least one honest person.*
3. *All used-car dealers are honest.*
4. *Everyone is both a used-car dealer and honest.*
5. *There is at least one honest used-car dealer (i.e. there is at least one person who is both a used-car dealer and honest).*
6. *There is at least one person who, if s/he is a used-car dealer, then s/he is honest.*
7. *All used-car dealers are dishonest.*

Exercise 6.2 *Expensive books*

1. $\exists X\,(b(X) \wedge e(X))$
2. $\forall X\,(b(X) \rightarrow e(X))$
3. $\neg\forall X\,(b(X) \rightarrow e(X))$ (or $\exists X\,(b(X) \wedge \neg e(X))$)
4. $\forall X\,(b(X) \rightarrow \exists Y\ a(Y,X))$
5. $b(ts) \rightarrow a(wg,\ ts)$
6. $a(wg,\ ts) \rightarrow b(ts)$

Exercise 6.3 *Numbers*

1. $\forall X\,(ratl(X) \rightarrow real(X))$

2. $\exists X\,(real(X) \wedge ratl(X))$

3. $\neg\forall X\,(real(X) \rightarrow ratl(X))$ (or $\exists X\,(real(X) \wedge \neg ratl(X))$)

Exercise 6.4 *More numbers*

1. *For every number, there is one and only one immediate successor. (That was my 'natural' paraphrase of the more literal translation: For every number, there is at least one number which is its immediate successor and if any other number is its immediate successor then that number isn't a different number.)*

2. *There is no natural number for which 0 is the immediate successor.*

3. *For every number other than 0, there is one and only one immediate predecessor.*

Exercise 6.5 *Evaluation w.r.t. a model (The following answers do not make all of my reasoning explicit, and are quite hard to read. Speak to me if this is causing any problems.)*

(1) False. The first conjunct is false because $[[a]]^M$ *is not in* $[[p]]^M$ *hence the wff is false.*

(2) True. $[[f(a)]]^M$ *is 2; the fist disjunct evaluates to false but the second is true because* $< [[a]]^M, [[f(a)]]^M > \in [[q]]^M$.

(3) True. If 1 is assigned to X*, the first conjunct is true (*$[[f]]^M$ *applied to 1 gives 2 which is in* $[[p]]^M$*); the second conjunct is also true (because* $[[f(a)]]^M$ *is 2 and* $<1, 2> \in [[q]]^M$*). Hence, the conjunct is true. There is no need to check the case where 2 is assigned to* X*. Since this was an existential, finding one value that gives us true is enough to conclude that the wff is true.*

(4) False. If 1 is assigned to X*, the first conjunct is false, so the whole conjunction will be false. If 2 is assigned to* X*, the first conjunct is true, but the second conjunct is false so the conjunction is false. We cannot find a value to assign to* X *to yield true, so the wff is false.*

(5) True. If 1 is assigned to X*, and 1 is assigned to* Y*, the atom is true. (There is no need to check the other assignment to* Y *because* Y *is existentially quantified and we have found an assignment that makes it true.)*

If 2 is assigned to X*, then if 1 is assigned to* Y*, the atom is false. But if 2 is assigned to* Y*, the atom is true. So the existential is true in this case too.*

Since the existential is true in all assignments to X*, the whole wff is true.*

(6) True. If 1 is assigned to Y*, and if 1 is assigned to* X*, the atom is true. But if 2 is assigned to* X*, the atom is false. So this assignment to* Y *doesn't make all assignments to* X *true.*

If 2 is assigned to Y and 1 is assigned to X, the atom is true. And if 2 is assigned to X the atom is true. So this assignment to Y does make all assignments to X true.

We conclude there as there is (at least) one assignment to Y that makes $\forall X\, q(X, y)$ true, then the whole wff is true.