MATHS FOR INFORMATION PROCESSING

Problem Sheet 5

1

(i) In the following strings, a is a constant symbol, s is a unary function symbol, u is a binary function symbol, e is a unary predicate symbol and g is a binary predicate symbol. Which of the strings are well-formed formulae of first-order predicate logic?

(a) $\forall x\, e(x)$

(b) $\exists y(g(a, y) \wedge e(x))$

(c) $\forall x \exists y(g(x, a))$

(d) $\forall x \exists y(g(x, a) \vee g(y, x))$

(e) $g(s(a, a), a)$

(f) $e(u(a, s(a))) \Rightarrow g(a, a)$

(g) $\exists x(u(a, x) \vee s(x))$

(h) $\forall x(e(x) \vee e(s(x)))$

(i) $\neg(e(x) \wedge e(s(x)))$

(j) $\forall x\, e(s(a))$

(k) $g(a, e(a))$

(l) $\forall x\, g(u(x, x), s(x)) \vee g(s(a), u(a, a))$

(ii) For each of the following wffs, indicate the scope of the quantifiers, their free variables (if any), and whether each is a sentence or not. (Parentheses are being used less liberally than normal, so you need to make use of the precedence rules.)

(a) $\exists x(p(x, y) \wedge q(x))$

(b) $\exists x\, p(x, y) \wedge q(x)$

(c) $\exists x \exists y\, p(x, y) \Rightarrow q(x)$

(d) $\exists x(\exists y\, p(x, y) \Rightarrow q(x))$

(e) $\neg\exists x \exists y\, p(x, y) \Rightarrow q(x)$

(f) $\forall x \neg\exists y\, p(x, y)$

(g) $\neg q(x) \Rightarrow (\neg\forall y(\neg p(x, y) \vee q(x)) \Rightarrow r(y))$

(h) $\exists x(q(x, y) \vee p(y))$

(i) $\exists x\, p(x, x) \vee \exists y\, q(y)$

(j) $\exists x(\exists y(p(x, y) \wedge q(y)) \Rightarrow \exists w\, r(x, w))$

(k) $\forall x(\forall y\, p(y, x) \Rightarrow q(y))$

(l) $\forall x \forall y\, p(x, y) \Rightarrow q(x)$

2

In this question, take the universe of discourse to be the set of natural numbers, $\mathbb{N}$. And here is $\mathcal{I}$:

$\mathcal{I}(a)$	is 0
$\mathcal{I}(b)$	is 1
$\mathcal{I}(c)$	is 3
$\mathcal{I}(f)$	is the unary function that maps a number to its square
$\mathcal{I}(g)$	is the binary function that maps a pair of numbers to their sum
$\mathcal{I}(p)$	is the unary relation that says whether a number is even or not
$\mathcal{I}(q)$	is the binary relation that says whether the first number divides the second number (i.e. does the first number 'go into' the second number a whole number of times with no remainder)

(i) What is the truth value of each of the following wffs according to this interpretation. Also express in English what each wff says.

(a) $p(a)$

(b) $p(b)$

(c) $p(c)$

(d) $p(f(a))$

(e) $p(f(b))$

(f) $p(f(c))$

(g) $p(g(a, b))$

(h) $p(g(a, c))$

(i) $p(g(b, c))$

(j) $p(g(f(b), c))$

(k) $p(g(f(c), f(b)))$

(l) $q(c, b)$

(m) $q(f(c), f(b))$

(n) $q(f(f(c)), f(f(b)))$

(o) $\forall x\, q(x, x)$

(p) $\forall x\, q(f(x), x)$

(q) $\exists x\, q(x, f(x))$

(r) $\forall x(p(x) \Rightarrow p(f(x)))$

(s) $\exists x(p(x) \wedge p(f(x)))$

(t) $\exists x \exists y(p(x) \wedge p(y) \wedge p(g(x, y)))$

(ii) Using the symbols from above, translate each of the following into first-order predicate logic. (The word 'number' should be read as 'natural number' throughout.)

(a) Every odd number divides its own square.

(b) Not every square divides the number it is the square of.

(c) If x divides y, then $x \uparrow 2$ divides $y \uparrow 2$.

(d) The square of an odd number is odd.

(e) If the sum of two numbers is even, so is the sum of their squares.

(f) There is an odd number x such that $x + 3$ divides $x + 9$

3

(i) Suppose the universe of discourse is defined as follows:

$$\mathcal{U} \triangleq \{1, 2\}$$

and that we have an interpretation of the constant symbols a and b and binary predicate symbol p as follows:

$\mathcal{I}(a)$ is 1

$\mathcal{I}(b)$ is 2

$\mathcal{I}(p)$ is the binary relation on $\mathcal{U}$ whose graph/extension is $\{\langle 1, 1\rangle, \langle 2, 2\rangle\}$

Determine the truth value of each of the following wffs. Show your working.

(a) $\forall x\, p(x, x)$

(b) $\exists y \neg p(a, y)$

(c) $\forall x \exists y\, p(x, y)$

(d) $\forall x \forall y\, p(x, y)$

(e) $\exists x \forall y\, p(x, y)$

(f) $\forall x \forall y (p(x, y) \Rightarrow p(y, x))$

(ii) Again the universe of discourse is the set $\{1, 2\}$. But we now have the following interpretation of the constant symbols a and b, the unary function symbol f and the binary predicate symbol p:

$\mathcal{I}(a)$ is 1

$\mathcal{I}(b)$ is 2

$\mathcal{I}(f)$ is the unary function on $\mathcal{U}$ whose maplets are $\{1 \mapsto 2, 2 \mapsto 1\}$

$\mathcal{I}(p)$ is the binary relation on $\mathcal{U}$ whose graph/extension is $\{\langle 1,1\rangle, \langle 1,2\rangle\}$

Evaluate the following wffs under the above interpretation. Show your working.

(a) $p(a, f(a)) \wedge p(b, f(b))$

(b) $\forall x \exists y\, p(y, x)$

(c) $\forall x \forall y (p(x, y) \Rightarrow p(f(x), f(y)))$

4

In this question, use the following 'key':

$$\begin{array}{rl} o: & \text{the empty set} \\ u(x,y): & \text{the union of } x \text{ and } y \\ i(x,y): & \text{the intersection of } x \text{ and } y \\ m(x,y): & x \text{ is a member of } y \\ s(x,y): & x \text{ is a subset of } y \\ e(x,y): & x \text{ is equal to } y \end{array}$$

(o is a constant symbol, u and i are binary function symbols, and m, s and e are binary predicate symbols.)

(i) What facts about sets are expressed by the following wffs?

(a) $\neg m(o, o)$

(b) $e(o, u(o, o))$

(c) $\forall x\, e(x, u(o, x))$

(d) $\exists x \forall y\, e(y, u(x, y))$

(e) $\forall x \forall y\, e(u(x,y), u(y,x))$

(f) $\forall x \forall y\, s(i(x,y), y)$

(ii) Translate the following statements into first-order predicate logic.

(a) The empty set is a subset of every set.

(b) Nothing is a member of the empty set.

(c) One set is a subset of another if and only if every member of the first is a member of the second.

(d) If one set is a subset of another, then there is a member of the second set that is not a member of the first.

(e) If something is a member of both of two sets then it is a member of the intersection of those sets.

(f) Any set is a subset of its union with any other set.

(iii) One of the statements you translated into first-order predicate logic in part (ii) above was false in set theory. Which one?

5

Use the following 'key' to translate the English statements into first-order predicate logic. (Where you think there are ambiguities, give multiple translations. Where you have problems in determining the exact meaning of the English, give the best translation(s) you can, and then say why your translation might deviate from the English.) Don't be discouraged if you find these hard. They are hard. (Some are much, much, much harder than anything you would be examined on.) At the very least, make sure that what you write down is a sentence of predicate logic.

$$\begin{aligned}
a &: \text{ Anne} \\
b &: \text{ Bill} \\
c &: \text{ Clare} \\
\\
ad(x, y) &: \ x \text{ admires } y \\
at(x, y) &: \ x \text{ attended } y \\
p(x) &: \ x \text{ is a practical} \\
s(x) &: \ x \text{ is a student} \\
l(x) &: \ x \text{ is a lecturer}
\end{aligned}$$

(i) Anne admires every lecturer.

(ii) Some lecturer admires Anne.

(iii) Anne admires herself.

(iv) No student attended every practical.

(v) No practical was attended by any student.

(vi) Bill admires anyone who admires him.

(vii) Bill admires anyone who admires him/herself.

(viii) Clare admires anyone who admires both Anne and Bill.

(ix) Nobody admires a lecturer.

(x) If nobody admires a lecturer, then no student attends practicals.

(xi) If any student attends practicals, Clare does.

(xii) If Anne attends a practical, Clare will also attend it.

(xiii) All lecturers, and some students, admire Anne.

(xiv) No student admires everything that some lecturers admire.

6 [Covers non-examinable material; Optional]

(i) In the two lists of wffs below, each wff on the left is equivalent to exactly one wff on the right. Find all the equivalent pairs. (Remind yourself of the precedence rules before proceeding.)

i.	$\exists x\, p(x) \wedge \exists x\, q(x)$	a.	$\exists y(p(y) \wedge \forall x\, r(x,y))$
ii.	$\exists x\, p(x) \wedge \forall y\, q(y)$	b.	$\forall z(p(z) \Rightarrow \exists u\, r(u,z))$
iii.	$\exists x(p(x) \wedge \forall y\, r(x,y))$	c.	$\exists x\, p(x) \wedge \exists y\, q(y)$
iv.	$\forall x(p(x) \Rightarrow \exists y\, r(x,y))$	d.	$\forall y \exists z\, r(y,z) \Rightarrow \exists w\, p(w)$
v.	$\forall x(p(x) \Rightarrow \exists y\, r(y,x))$	e.	$\exists u(p(u) \wedge \forall w\, r(u,w))$
vi.	$\forall y(\exists z\, r(y,z) \Rightarrow p(y))$	f.	$\exists x\, p(z) \wedge \forall z\, q(z)$
vii.	$\exists x(p(x) \wedge \forall y\, r(y,x))$	g.	$\forall x(\exists y\, r(x,y) \Rightarrow p(x))$
viii.	$\forall x \exists y\, r(x,y) \Rightarrow \exists x\, p(x)$	h.	$\forall x(p(x) \Rightarrow \exists z\, r(x,z))$

(ii) Use algebraic manipulation to show

(a) $\forall x(p(x) \Rightarrow \exists y(q(y) \wedge r(x,y))) \equiv \neg\exists x(p(x) \wedge \forall y(q(y) \Rightarrow \neg r(x,y)))$

(b) $\forall x \forall y(p(x,y) \Rightarrow p(y,x)) \equiv \neg\exists x \exists y(p(x,y) \wedge \neg p(y,x))$

Model Answers

Maths for Information Processing

Problem Sheet 5

Question 1

Part (i)
Option (a)

Yes

Option (b)

Yes: that x is free is not a problem

Option (c)

Yes: that y is not used is not a problem

Option (d)

Yes

Option (e)

No: s is a unary function symbol but appears here with two arguments

Option (f)

Yes

Option (g)

No: s and u are function symbols but appear here in predicate symbol positions

Option (h)

Yes

Option (i)

Yes: that x is free is not a problem

Option (j)

Yes: that x is not used is not a problem

Option (k)

No: e is a predicate symbol but appears here in a function symbol position

Option (l)

Yes

Part (ii)
Option (a)

Scope: $\underline{\exists x(p(x,y) \wedge q(x))}$

Free: $\exists x(p(x,\underline{y}) \wedge q(x))$

Not a sentence

Option (b)

Scope: $\underline{\exists x\, p(x, y)} \wedge q(x)$

Free: $\exists x\, p(x, \underline{y}) \wedge q(\underline{x})$

Not a sentence

Option (c)

Scope: $\underline{\exists x \underline{\exists y\, p(x, y)}} \Rightarrow q(x)$

Free: $\exists x \exists y\, p(x, y) \Rightarrow q(\underline{x})$

Not a sentence

Option (d)

Scope: $\underline{\exists x(\underline{\exists y\, p(x, y)} \Rightarrow q(x))}$

Free: None

A sentence

Option (e)

Scope: $\neg \underline{\exists x \underline{\exists y\, p(x, y)}} \Rightarrow q(x)$

Free: $\neg \exists x \exists y\, p(x, y) \Rightarrow q(\underline{x})$

Not a sentence

Option (f)

Scope: $\underline{\forall x \neg \underline{\exists y\, p(x, y)}}$

Free: None

A sentence

Option (g)

Scope: $\neg q(x) \Rightarrow (\neg \underline{\forall y(\neg p(x, y) \vee q(x))} \Rightarrow r(y))$

Free: $\neg q(\underline{x}) \Rightarrow (\neg \forall y(\neg p(\underline{x}, y) \vee q(\underline{x})) \Rightarrow r(\underline{y}))$

Not a sentence

Option (h)

Scope: $\underline{\exists x(q(x, y) \vee p(y))}$

Free: $\exists x(q(x, \underline{y}) \vee p(\underline{y}))$

Not a sentence

Option (i)

Scope: $\underline{\exists x\, p(x, x)} \vee \underline{\exists y\, q(y)}$

Free: None

A sentence

Option (j)

Scope: $\underline{\exists x(\underline{\exists y(p(x,y) \wedge q(y))} \Rightarrow \underline{\exists w\, r(x,w)})}$

Free: None

A sentence

Option (k)

Scope: $\underline{\forall x(\underline{\forall y\, p(y,x)} \Rightarrow q(y))}$

Free: $\forall x(\forall y\, p(y,x) \Rightarrow q(\underline{y}))$

Not a sentence

Option (l)

Scope: $\underline{\forall x\underline{\forall y\, p(x,y)}} \Rightarrow q(x)$

Free: $\forall x\forall y\, p(x,y) \Rightarrow q(\underline{x})$

Not a sentence.

Question 2

Part (i)

Option (a)

0 is even; true

Option (b)

1 is even; false

Option (c)

3 is even; false

Option (d)

The square of 0 is even; true

Option (e)

The square of 1 is even; false

Option (f)

The square of 3 is even; false

Option (g)

The sum of 0 and 1 is even; false

Option (h)

The sum of 0 and 3 is even; false

Option (i)

The sum of 1 and 3 is even; true

Option (j)

The sum of the square of 1, and 3, is even; true

Option (k)

The sum of the squares of 3 and 1 is even; true

Option (l)

3 divides 1; false

Option (m)

The square of 3 divides the square of 1; false

Option (n)

The square of the square of 3 divides the square of the square of 1; false

Option (o)

Every natural number divides itself; false (true for all natural numbers except 0)

Option (p)

The square of every natural number divides that natural number; false

Option (q)

Some natural number divides its square; true

Option (r)

Every even natural number has an even square; true

Option (s)

Some even natural number has an even square; true

Option (t)

There are at least two even natural numbers whose sum is even; true

Part (ii)

Option (a)

$\forall x(\neg p(x) \Rightarrow q(x, f(x)))$

Option (b)

$\neg\forall x\, q(f(x), x)$

Option (c)

$\forall x\forall y(q(x, y) \Rightarrow q(f(x), f(y)))$

Option (d)

$\forall x(\neg p(x) \Rightarrow \neg p(f(x)))$

Option (e)

$\forall x\forall y(p(g(x, y)) \Rightarrow p(g(f(x), f(y))))$

Option (f)

$\exists x(\neg p(x) \wedge q(g(x, c), g(x, f(c))))$

Question 3

Part (i)
Option (a)

- Take $\mathcal{I}(x)$ to be 1.
  - $\langle 1, 1\rangle \in \mathcal{I}(p)$
- Take $\mathcal{I}(x)$ to be 2.
  - $\langle 2, 2\rangle \in \mathcal{I}(p)$

The wff as a whole is true.
Option (b)

- Take $\mathcal{I}(y)$ to be 1.
  - We know $\mathcal{I}(a)$ is 1, $\langle 1, 1\rangle \in \mathcal{I}(p)$, so $\mathcal{I}(p(a, y))$ is true. Its negation is therefore false. So we must try another assignment.
- Take $\mathcal{I}(y)$ to be 2.
  - $\langle 1, 2\rangle \notin \mathcal{I}(p)$. So $\mathcal{I}(p(a, y))$ is false. Therefore its negation is true.

So there is an assignment to y that makes $p(a, y)$ true. So the wff as a whole is true.
Option (c)

- Take $\mathcal{I}(x)$ to be 1.
  - Take $\mathcal{I}(y)$ to be 1.
    * $\langle 1, 1\rangle \in \mathcal{I}(p)$

    So the existential is true.
- Take $\mathcal{I}(x)$ to be 2.
  - Take $\mathcal{I}(y)$ to be 1.
    * $\langle 1, 2\rangle \notin \mathcal{I}(p)$
  - Take $\mathcal{I}(y)$ to be 2.
    * $\langle 2, 2\rangle \in \mathcal{I}(p)$

    So the existential is true.

The existential is true for all assignments to x, so the wff as a whole is true.

Option (d)

The answer to the previous questions shows that this is false: as that answer shows, when $\mathcal{I}(x)$ is 1 and $\mathcal{I}(y)$ is 2, $\langle 1, 2\rangle \notin \mathcal{I}(p)$. This means that $p(x, y)$ isn't true for all assignments to y. So the wff as a whole is false.

Option (e)

From the answer to the previous question, we know that $p(x, y)$ is not true for all y when x is 1. So there's no point trying that assignment to x again.

- Take $\mathcal{I}(x)$ to be 2.
  - Take $\mathcal{I}(y)$ to be 1.
    - $\langle 2, 1\rangle \notin \mathcal{I}(p)$

    As there is an assignment to y that fails to make $p(x, y)$ true, the universal is false.

So there is no assignment to x for which all assignments to y make $p(x, y)$ true. So the wff as a whole is false.

Option (f)

- Take $\mathcal{I}(x)$ to be 1.
  - Take $\mathcal{I}(y)$ to be 1.
    - The antecedent is true ($\langle 1, 1\rangle \in \mathcal{I}(p)$), and the consequent is true (ditto), so the conditional is true.
  - Take $\mathcal{I}(y)$ to be 2.
    - The antecedent is false ($\langle 1, 2\rangle \notin \mathcal{I}(p)$), so the conditional is true.
- Take $\mathcal{I}(x)$ to be 2.
  - Take $\mathcal{I}(y)$ to be 1.
    - The antecedent is false ($\langle 2, 1\rangle \notin \mathcal{I}(p)$), so the conditional is true.
  - Take $\mathcal{I}(y)$ to be 2.
    - The antecedent is true ($\langle 2, 2\rangle \in \mathcal{I}(p)$), and so is the consequent (ditto), so the conditional is true.

The wff as a whole is true.

Part (ii)

Option (a)

$\mathcal{I}(a)$ is 1. $\mathcal{I}(f(a))$ is 2. $\langle 1, 2\rangle \in \mathcal{I}(p)$, so the first conjunct is true.

$\mathcal{I}(b)$ is 2. $\mathcal{I}(f(b))$ is 1. $\langle 2, 1\rangle \notin \mathcal{I}(p)$, so the second conjunct is false.

The wff as a whole is false.

Option (b)

- Take $\mathcal{I}(x)$ to be 1.
  - Take $\mathcal{I}(y)$ to be 1.
    * $\langle 1, 1\rangle \in \mathcal{I}(p)$

    So at least one assignment to y makes $p(y, x)$ true when x is 1.
- Take $\mathcal{I}(x)$ to be 2.
  - Take $\mathcal{I}(y)$ to be 1.
    * $\langle 1, 2\rangle \in \mathcal{I}(p)$

    So at least one assignment to y makes $p(y, x)$ true when x is 2.

So $\exists y\, p(y, x)$ is true for all assignments to x, and so the wff as a whole is true.

Option (c)

- Take $\mathcal{I}(x)$ to be 1.
  - Take $\mathcal{I}(y)$ to be 1.
    * The antecedent is true ($\langle 1, 1\rangle \in \mathcal{I}(p)$), but the consequent is false($\mathcal{I}(f(x))$ and $\mathcal{I}(f(y))$ are both 2 and $\langle 2, 2\rangle \notin \mathcal{I}(p)$), so the conditional is false.

So already we see that the universal quantification of y is not true for this assignment to x. So the whole wff is false.

Question 4

Part (i)

Option (a)

The empty set is not a member of itself, i.e. $\emptyset \notin \emptyset$

Option (b)

The union of the empty set with itself is the empty set, i.e. $\emptyset = \emptyset \cup \emptyset$

Option (c)

Every set is equal to the union of itself with the empty set, i.e. $\emptyset = \emptyset \cup A$ for any set A

Option (d)

There is a set that when unioned with any set y gives y, i.e. there is an identity element for union

Option (e)

Set union is commutative

Option (f)

The intersection of two sets x and y is a subset of set y

Part (ii)

Option (a)

$\forall x\ s(o, x)$

Option (b)

$\neg \exists x\ m(x, o)$

Option (c)

$\forall x \forall y (s(x, y) \Leftrightarrow \forall z (m(z, x) \Rightarrow m(z, y)))$

Option (d)

$\forall x \forall y (s(x, y) \Rightarrow \exists z (m(z, y) \wedge \neg m(z, x)))$

Option (e)

$\forall x \forall y \forall z ((m(z, x) \wedge m(z, y)) \Rightarrow m(z, i(x, y)))$

Option (f)

$\forall x \forall y\ s(x, u(x, y))$

Part (iii)

(d) was false: it would be true as a part of the definition of *proper subset*, but not of subset.

Question 5

Part (i)

$\forall x (l(x) \Rightarrow ad(a, x))$

Part (ii)

$\exists x (l(x) \wedge ad(x, a))$

Part (iii)

$ad(a, a)$

Part (iv)

$\neg\exists x(s(x) \wedge \forall y(p(y) \Rightarrow at(x,y)))$. There are variants, e.g. $\forall x\neg(s(x) \wedge \forall y(p(y) \Rightarrow at(x,y)))$, $\forall x\neg(s(x) \Rightarrow \neg\forall y(p(y) \Rightarrow at(x,y)))$ and $\forall x\neg(s(x) \Rightarrow \exists y(p(y)\wedge\neg at(x,y)))$

Part (v)

I take this to mean that nothing at all was attended. $\neg\exists x(p(x)\wedge\exists y(s(y)\wedge at(y,x)))$ and variants of this

Part (vi)

$\forall x(ad(x,b) \Rightarrow ad(b,x))$

Part (vii)

$\forall x(ad(x,x) \Rightarrow ad(b,x))$

Part (viii)

$\forall x((ad(x,a) \wedge ad(x,b)) \Rightarrow ad(c,x))$

Part (ix)

This has a scope ambiguity: $\exists x(l(x) \wedge \forall y\neg ad(y,x))$ or $\forall y\exists x(l(x) \wedge \neg ad(y,x))$

Part (x)

I take the word 'practicals' to be universally quantified: no practicals are attended. The antecedent has the same scope ambiguity as the one in the previous question, so we get $(\exists x(l(x) \wedge \forall y\neg ad(y,x))) \Rightarrow \neg\exists x(p(x) \wedge \exists y(s(y) \wedge ay(y,x)))$ or $(\forall y\exists x(l(x) \wedge \neg ad(y,x))) \Rightarrow \neg\exists x(p(x) \wedge \exists y(s(y) \wedge ay(y,x)))$

Part (xi)

Again, I take 'practicals' to be universally quantified: $(\exists x(s(x) \wedge \forall y(p(y) \Rightarrow at(x,y)))) \Rightarrow \forall z(p(z) \Rightarrow at(c,z))$

Part (xii)

$\forall x((p(x) \wedge at(a,x)) \Rightarrow at(c,x))$

Part (xiii)

$\forall x(l(x) \Rightarrow ad(x,a)) \wedge \exists y(s(y) \wedge ad(y,a))$

Part (xiv)

There is a scope ambiguity here: $\exists x(l(x) \wedge \forall y(ad(x,y) \Rightarrow \neg\exists z(s(z) \wedge ad(z,y))))$ or $\forall z(s(z) \Rightarrow \exists x(l(x) \wedge \forall y(ad(x,y) \Rightarrow \neg ad(z,y))))$

Question 6 [Covers non-examinable material; Optional]

Part (i)

i with c; ii with f; iii with e; iv with h; v with b; vi with g; vii with a; viii with d

Part (ii)
Option (a)

$$
\begin{aligned}
\text{LHS} &\equiv \forall x(p(x) \Rightarrow \exists y(q(y) \wedge r(x,y))) \\
&\equiv \neg\exists x\neg(p(x) \Rightarrow \exists y(q(y) \wedge r(x,y))) && \text{(defn of existential)} \\
&\equiv \neg\exists x\neg(\neg p(x) \vee \exists y(q(y) \wedge r(x,y))) && \text{(defn of conditional)} \\
&\equiv \neg\exists x(\neg\neg p(x) \wedge \neg\exists y(q(y) \wedge r(x,y))) && \text{(De Morgan)} \\
&\equiv \neg\exists x(p(x) \wedge \neg\exists y(q(y) \wedge r(x,y))) && \text{(involution)} \\
&\equiv \neg\exists x(p(x) \wedge \neg\neg\forall y\neg(q(y) \wedge r(x,y))) && \text{(defn of universal)} \\
&\equiv \neg\exists x(p(x) \wedge \forall y\neg(q(y) \wedge r(x,y))) && \text{(involution)} \\
&\equiv \neg\exists x(p(x) \wedge \forall y(\neg q(y) \vee \neg r(x,y))) && \text{(De Morgan)} \\
&\equiv \neg\exists x(p(x) \wedge \forall y(q(y) \Rightarrow \neg r(x,y))) && \text{(defn of conditional)} \\
&\equiv \text{RHS}
\end{aligned}
$$

Option (b)

$$
\begin{aligned}
\text{LHS} &\equiv \forall x\forall y(p(x,y) \Rightarrow p(y,x)) \\
&\equiv \neg\exists x\neg\forall y(p(x,y) \Rightarrow p(y,x)) && \text{(defn of universal)} \\
&\equiv \neg\exists x\neg\neg\exists y\neg(p(x,y) \Rightarrow p(y,x)) && \text{(defn of universal)} \\
&\equiv \neg\exists x\exists y\neg(p(x,y) \Rightarrow p(y,x)) && \text{(involution)} \\
&\equiv \neg\exists x\exists y\neg(\neg p(x,y) \vee p(y,x)) && \text{(defn of conditional)} \\
&\equiv \neg\exists x\exists y(\neg\neg p(x,y) \wedge \neg p(y,x)) && \text{(De Morgan)} \\
&\equiv \neg\exists x\exists y(p(x,y) \wedge \neg p(y,x)) && \text{(involution)} \\
&\equiv \text{RHS}
\end{aligned}
$$