ITBML Mock Examinations 1996-97

Department of Computer Science

Part Ia

Mathematics for Information Processing

Time allowed: **30 minutes**

Silent non-programmable calculators may be used.
A set of 'Examination Notes' is attached. (*Actually they aren't. But they will be for the real exam.*)
Candidates should answer **all** questions from this paper.

1 (15 marks)

(i) [4 marks] In this part of the question,

$$\begin{aligned} p_1 &\triangleq \text{My program compiles} \\ p_2 &\triangleq \text{My program runs} \\ p_3 &\triangleq \text{There is a bug in the Pascal compiler} \\ p_4 &\triangleq \text{There is a system fault} \end{aligned}$$

(a) [2 marks] Give natural English paraphrases for the following well-formed formulae of propositional logic:

- $(p_2 \vee p_3) \wedge \neg(p_2 \wedge p_3)$
- $p_2 \Leftrightarrow (\neg p_3 \wedge \neg p_4)$

(b) [2 marks] Express the following statements as *well-formed formulae of propositional logic*:

- My program compiles only if there isn't a bug in the Pascal compiler.
- My program compiles if there isn't a bug in the Pascal compiler.

(ii) [3 marks] Draw the **full** *truth-tables* of the following well-formed formulae of propositional logic. Hence, state whether each is *valid* or *invalid*, and *satisfiable* or *unsatisfiable*.

(a) [1 mark] $(p \Rightarrow p) \Rightarrow p$

(b) [2 marks] $((p \vee q) \wedge (p \vee \neg q)) \wedge r$

(iii) [8 marks]

(a) [3 marks] Use a *truth-table* to show that

$$\{p \Rightarrow q, q \Rightarrow r\} \models p \Rightarrow r$$

(b) [4 marks] Use the rules of the *natural deduction* system given in the 'Examination Notes' to show that

$$\{p \Rightarrow q, q \Rightarrow r\} \vdash p \Rightarrow r$$

(Hint: you will require a subderivation.)

(c) [1 mark] Write a brief comment on the relationship between logical consequence ($\models$) and derivation ($\vdash$).

Model Answers

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Model Answers

Question 1 (15 marks)

Part (i) [4 marks]

Sub-part (a) [2 marks]

- My program runs unless there is a bug in the Pascal compiler. ('unless' is the most natural translation of exclusive-or.)
- My program runs precisely when/if and only if there isn't a bug in the Pascal compiler and there isn't a system fault.

Sub-part (b) [2 marks]

- $p_1 \Rightarrow \neg p_3$
- $\neg p_3 \Rightarrow p_1$

Part (ii) [3 marks]

Sub-part (a) [1 mark]

p	$(p$	$\Rightarrow$	$p)$	$\Rightarrow$	p
T	T	T	T	T	T
F	F	T	F	F	F
				$\triangle$	

Invalid but satisfiable

Sub-part (b) [2 marks]

p	q	r	$((p$	$\vee$	$q)$	$\wedge$	$(p$	$\vee$	$\neg$	$q))$	$\wedge$	r
T	T	T	T	T	T	T	T	T	F	T	T	T
T	T	F	T	T	T	T	T	T	F	T	F	F
T	F	T	T	T	F	T	T	T	T	F	T	T
T	F	F	T	T	F	T	T	T	T	F	F	F
F	T	T	F	T	T	F	F	F	F	T	F	T
F	T	F	F	T	T	F	F	F	F	T	F	F
F	F	T	F	F	F	F	F	T	T	F	F	T
F	F	F	F	F	F	F	F	T	T	F	F	F
											$\triangle$	

Invalid but satisfiable

Part (iii) [8 marks]

Sub-part (a) [3 marks]

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$p \Rightarrow r$	
T	T	T	T	T	T	*1
T	T	F	T	F	F	
T	F	T	F	T	T	
T	F	F	F	T	F	
F	T	T	T	T	T	*2
F	T	F	T	F	T	
F	F	T	T	T	T	*3
F	F	F	T	T	T	*4

In all the interpretations in which all of the premisses are true (rows marked *1, *2, *3 and *4), the conclusion is also true.

Sub-part (b) [4 marks]

1.	$p \Rightarrow q$	premiss
2.	$q \Rightarrow r$	premiss
3.	Subderivation	
3.1	p	assumption
3.2	q	3.1,1,$\Rightarrow$-E
3.3	r	3.2,2,$\Rightarrow$-E
4.	$p \Rightarrow r$	3,$\Rightarrow$-I

Sub-part (c) [1 mark]

For a sound and complete proof-theory, $A \models W$ if and only if $A \vdash W$