ITBML Mock Examinations 1996-97

Department of Computer Science

Part Ia

Mathematics for Information Processing

Time allowed: **30 minutes**

Silent non-programmable calculators may be used.
A set of 'Examination Notes' is attached. (*Actually they aren't. But they will be for the real exam.*)
Candidates should answer **all** questions from this paper.

1 (5 marks)

In this part of the question,

$$\begin{aligned} S &\triangleq \{a, b, c\} \\ R_1 &: S \leftrightarrow S \\ R_1 &\triangleq \{\langle b, a\rangle, \langle c, b\rangle\} \\ R_2 &: S \leftrightarrow S \\ R_2 &\triangleq \{\langle a, c\rangle, \langle b, c\rangle, \langle c, c\rangle\} \\ R_3 &: S \leftrightarrow S \\ R_3 &\triangleq \{\langle a, b\rangle, \langle c, a\rangle\} \\ R_4 &: S \leftrightarrow S \\ R_4 &\triangleq R_1 \cup R_2 \cup R_3 \end{aligned}$$

(a) [3 marks] Which, if any, of the four relations are bijections? **Briefly justify** your answer.

(b) [2 marks] Is R_4 an *order*? If you think it is, state whether it is strict or non-strict and whether it is a total order or a partial order. If you think it is not, explain why it is not.

2 (7 marks)

In this question, S is a set of software engineers, P is a set of programming projects, and L is a set of programming languages. *willUse* is a relation from projects to languages that says which projects will use which programming languages; *canUse* is a relation from software engineers to languages that says which engineers can program in which languages. This is stated more formally below. (Only a few members of the sets and relation graphs are shown.)

$$\begin{aligned} S &\triangleq \{\mathit{Ann},\ \mathit{Bill}, \ldots\} \\ P &\triangleq \{\mathit{Proj23},\ \mathit{Proj211},\ \mathit{Proj14}, \ldots\} \\ L &\triangleq \{\mathit{Pascal},\ \mathit{C{+}{+}}, \ldots\} \end{aligned}$$

$$\begin{aligned} \mathit{willUse} &: P \leftrightarrow L \\ \mathit{willUse} &\triangleq \{\langle \mathit{Proj23},\ \mathit{Pascal}\rangle,\ \langle \mathit{Proj211},\ \mathit{C{+}{+}}\rangle, \ldots\} \end{aligned}$$

$$\begin{aligned} \mathit{canUse} &: S \leftrightarrow L \\ \mathit{canUse} &\triangleq \{\langle \mathit{Ann},\ \mathit{Pascal}\rangle,\ \langle \mathit{Ann},\ \mathit{C{+}{+}}\rangle, \ldots\} \end{aligned}$$

Write expressions involving these relations that, when evaluated, would compute:

(a) [1 mark] the language(s) that Bill can use;

(b) [1 mark] the software engineers that can use the language COBOL;

(c) [$2\frac{1}{2}$ marks] whether Bill is suited to Proj23 (in terms of knowing one of more of the needed programming languages);

(d) [$2\frac{1}{2}$ marks] whether there are projects that use languages for which we have no software engineers.

3 (3 marks)

Recall that $\mathbb{Z}$ is the set of integers (negative, zero and positive) and $\mathbb{N}$ is the set of natural numbers (zero and positive integers).

Let the functions f_1 and f_2 be defined as follows:

$$\begin{aligned} f_1 &: \mathbb{Z} \rightarrow \mathbb{N} \\ f_1(x) &\triangleq x \uparrow 2 \\ f_2 &: \mathbb{N} \rightarrow \mathbb{Z} \\ f_2(x) &\triangleq 1/x \end{aligned}$$

(a) [$1\frac{1}{2}$ marks]

- State whether f_1 is *total* or *partial*. **Briefly justify** your answer.

- State whether f_2 is *total* or *partial*. **Briefly justify** your answer.

(b) [$1\frac{1}{2}$ marks] The function f_3 is equal to the *composition* $f_1 \circ f_2$. Give a definition of f_3. PROBLEM: gives equiv anasesr for either ordering of composition, so change it.

Model Answers

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Model Answers

Question 1 (5 marks)

Sub-part (a) [3 **marks**]

None are. We require total, surjective, functional and injective. So, to show that each is not a bijection, I need only show that, for each relation, one of these four properties fails to hold.

R_1 is not total: it does not map a to anything. R_2 is not injective: it maps two things to c. R_3 is not total: it does not map b to anything. R_4 is not functional: it maps a to more than one thing. [Other justifications are possible.]

Sub-part (b) [**2 marks**]

It is not. It must be antisymmetric and transitive.

It is transitive.

But it is not antisymmetric: e.g, in the graph of R_4, from R_2 we get $\langle a, c \rangle$, and from R_3 we get $\langle c, a \rangle$.

(This last part is all that is needed for a correct answer, since all one must show is that one of the two properties doesn't hold.)

Question 2 (7 marks)

Sub-part (a) [1 mark]

$Im(canUse, \{Bill\})$

Sub-part (b) [1 mark]

$Im(canUse^{-1}, \{COBOL\})$

Sub-part (c) [$2\frac{1}{2}$ marks]

$Proj23 \in Im(willUse^{-1} \circ canUse, \{Bill\})$
or $\langle Bill, Proj23\rangle \in (willUse^{-1} \circ canUse)$
or $(Im(canUse, \{Bill\}) \cap Im(willUse, \{Proj23\})) \neq \emptyset$

Sub-part (d) [$2\frac{1}{2}$ marks]

$Ran(willUse) \setminus Ran(canUse) \neq \emptyset$

Question 3 (3 marks)

Sub-part (a) [$1\frac{1}{2}$ **marks**]

f_1 is total: every integer has a natural square; f_2 is partial: only 1 is mapped (nothing else has an integer reciprocal).

Sub-part (b) [$1\frac{1}{2}$ **marks**]

$$f_3 : \mathbb{N} \to \mathbb{N}$$

$$f_3(x) \triangleq (1/x) \uparrow 2$$