B.Sc. and B.A. Mock Examinations 1996-97

Department of Computer Science

Part Ia

Mathematics for Information Processing

Time allowed: **30 minutes**

Silent non-programmable calculators may be used.
A set of 'Examination Notes' is attached. (*Actually they aren't. But they will be for the real exam.*)
Candidates should answer **all** questions from this paper.

1 (5 marks)

The following set names are used in parts of this question:

$\mathbb{N}$: the set of natural numbers (positive integers and zero)
$\mathbb{P}$: the set of positive integers (excludes zero)
$\mathbb{Z}$: the set of all integers (negative, zero and positive)

Evaluate the following expressions:

(a) [1 mark]
$(\{x \mid x \in \mathbb{N} \text{ and } x > 2 \text{ and } x < 6\} \cup \{x \mid x \in \mathbb{P} \text{ and } x \geq 5 \text{ and } x \leq 8\}) \setminus (\{1,2,3\} \cap \{3,2,1\})$

(b) [1 mark] $\{1,2\} \subseteq \{\{1,2\}\}$

(c) [1 mark] $\{1,1\} \in \{\{1\},\{2\},\{3\}\}$

(d) [1 mark] $\mathit{card}(\{6\}) < \mathit{card}(\{2,4\})$

(e) [1 mark] $\mathit{card}(\wp(\mathbb{P} \cap (\mathbb{Z} \setminus \mathbb{N})))$

2 (5 marks)

In this question,

$$\begin{aligned} U &\triangleq \{b \mid b \text{ is a book}\} \\ A &\triangleq \{b \mid b \text{ is a hardback book}\} \\ B &\triangleq \{b \mid b \text{ is a book with a blue cover}\} \end{aligned}$$

Write expressions that, when evaluated, would compute

(a) [1 mark] the set of blue hardback books;

(b) [1 mark] the set of blue paperback books;

(c) [1 mark] the set of books that are neither blue nor hardback;

(d) [1 mark] how many paperback books there are; and

(e) [1 mark] whether there are any blue paperback books.

3 (5 marks)

In this question, use the laws given in the 'Examination Notes' and the following additional law: for any sets SE_1 and SE_2,

$$SE_1 \setminus SE_2 = SE_1 \cap SE_2'$$

Prove the following algebraically: for any sets A_1 and A_2,

$$(A_1 \cup A_2)' \setminus (A_1 \cap A_2) = A_1' \cap A_2'$$

You **must** annotate each step of your proof with the algebraic law that is used in that step.

Model Answers

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Model Answers

Question 1 (5 marks)

Sub-part (a) [1 mark]

$\{4, 5, 6, 7, 8\}$

Sub-part (b) [1 mark]

false

Sub-part (c) [1 mark]

true

Sub-part (d) [1 mark]

true

Sub-part (e) [1 mark]

$= card(\wp(\emptyset)) = card(\{\emptyset\}) = 1$

Question 2 (5 marks)

Sub-part (a) [1 mark]

$A \cap B$

Sub-part (b) [1 mark]

$A' \cap B$ or $B \setminus A$

Sub-part (c) [1 mark]

$(A \cup B)'$ or $A' \cap B'$ or $U \setminus (A \cup B)$, etc.

Sub-part (d) [1 mark]

$card(A')$

Sub-part (e) [1 mark]

$(A' \cap B) \neq \emptyset$ or $card(A' \cap B) > 0$, etc.

Question 3 (5 marks)

$$
\begin{aligned}
\text{LHS} &= (A_1 \cup A_2)' \setminus (A_1 \cap A_2) \\
&= (A_1 \cup A_2)' \cap (A_1 \cap A_2)' && \text{(defn of set diff)} \\
&= (A_1' \cap A_2') \cap (A_1 \cap A_2)' && \text{(De Morgan)} \\
&= (A_1' \cap A_2') \cap (A_1' \cup A_2') && \text{(De Morgan)} \\
&= ((A_1' \cap A_2') \cap A_1') \cup ((A_1' \cap A_2') \cap A_2') && \text{(distrib of } \cap \text{ over } \cup\text{)} \\
&= (A_1' \cap (A_1' \cap A_2')) \cup ((A_1' \cap A_2') \cap A_2') && \text{(commutativity of } \cap\text{)} \\
&= ((A_1' \cap A_1') \cap A_2') \cup ((A_1' \cap A_2') \cap A_2') && \text{(assoc of } \cap\text{)} \\
&= (A_1' \cap A_2') \cup ((A_1' \cap A_2') \cap A_2') && \text{(idempotence)} \\
&= (A_1' \cap A_2') \cup (A_1' \cap (A_2' \cap A_2')) && \text{(assoc of } \cap\text{)} \\
&= (A_1' \cap A_2') \cup (A_1' \cap A_2') && \text{(idempotence)} \\
&= (A_1' \cap A_2') && \text{(idempotence)} \\
&= \text{RHS}
\end{aligned}
$$