MScIP Mock Examinations 1996-97

Department of Computer Science

Part MSca

Mathematics for Information Processing

Time allowed: **30 minutes**

Silent non-programmable calculators may be used.
A set of 'Examination Notes' is attached. (*Actually they aren't. But they will be for the real exam.*)
Candidates should answer **all** questions from this paper.

1 (15 marks)

(i) [4 marks] In this part of the question,

$$
\begin{aligned}
p_1 &\triangleq \text{My program compiles} \\
p_2 &\triangleq \text{My program runs} \\
p_3 &\triangleq \text{There is a bug in the Pascal compiler} \\
p_4 &\triangleq \text{There is a system fault}
\end{aligned}
$$

(a) [2 marks] Give natural English paraphrases for the following well-formed formulae of propositional logic:

- $\neg(p_3 \vee p_4) \Rightarrow (p_1 \Rightarrow p_2)$
- $(\neg p_3 \vee \neg p_4) \Rightarrow (p_2 \Leftrightarrow p_1)$

(b) [2 marks] Express the following statements as *well-formed formulae of propositional logic*:

- There's a system fault precisely when my program compiles but doesn't run.
- My program compiles and runs unless there's a bug in the Pascal compiler.

(ii) [3 marks] Draw the **full** *truth-tables* of the following well-formed formulae of propositional logic. Hence, state whether each is *valid* or *invalid*, and *satisfiable* or *unsatisfiable*.

(a) [1 mark] $(p \Leftrightarrow p) \Leftrightarrow (p \Leftrightarrow p)$

(b) [2 marks] $((p \wedge q) \vee (p \wedge \neg q)) \vee r$

(iii) [8 marks]

(a) [3 marks] Use a *truth-table* to show that

$$\{p \Leftrightarrow q, q \Leftrightarrow r\} \models p \Rightarrow r$$

(b) [4 marks] Use the rules of the *natural deduction* system given in the 'Examination Notes' to show that

$$\{p \Leftrightarrow q, q \Leftrightarrow r\} \vdash p \Rightarrow r$$

(Hint: you will require a subderivation.)

(c) [1 mark] Write a brief comment on the relationship between logical consequence ($\models$) and derivation ($\vdash$).

Model Answers

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Model Answers

Question 1 (15 marks)

Part (i) [4 marks]

Sub-part (a) [2 marks]

- If there's neither a compiler bug nor a system fault, then if my program compiles it runs.
- If either there isn't a compiler bug or there isn't a system fault, then my program will run exactly if it compiles.

Sub-part (b) [2 marks]

- $p_4 \Leftrightarrow (p_1 \wedge \neg p_2)$
- $((p_1 \wedge p_2) \vee p_3) \wedge \neg((p_1 \wedge p_2) \wedge p_3)$

Part (ii) [3 marks]

Sub-part (a) [1 mark]

p	$(p$	$\Leftrightarrow$	$p)$	$\Leftrightarrow$	$(p$	$\Leftrightarrow$	$p)$
T	T	T	T	T	T	T	T
F	F	T	F	T	F	T	F
				$\triangle$			

Valid and satisfiable

Sub-part (b) [2 marks]

p	q	r	$((p$	$\wedge$	$q)$	$\vee$	$(p$	$\wedge$	$\neg$	$q))$	$\vee$	r
T	T	T	T	T	T	T	T	F	F	T	T	T
T	T	F	T	T	T	T	T	F	F	T	T	F
T	F	T	T	F	F	T	T	T	T	F	T	T
T	F	F	T	F	F	T	T	T	T	F	T	F
F	T	T	F	F	T	F	F	F	F	T	T	T
F	T	F	F	F	T	F	F	F	F	T	F	F
F	F	T	F	F	F	F	F	F	T	F	T	T
F	F	F	F	F	F	F	F	F	T	F	F	F
											$\triangle$	

Invalid but satisfiable

Part (iii) [8 marks]

Sub-part (a) [3 marks]

p	q	r	$p \Leftrightarrow q$	$q \Leftrightarrow r$	$p \Rightarrow r$	
T	T	T	T	T	T	*1
T	T	F	T	F	F	
T	F	T	F	F	T	
T	F	F	F	T	F	
F	T	T	F	T	T	
F	T	F	F	F	T	
F	F	T	T	F	T	
F	F	F	T	T	T	*2

In all the interpretations in which all of the premisses are true (rows marked *1 and *2), the conclusion is also true.

Sub-part (b) [4 marks]

1. $p \Leftrightarrow q$ premiss
2. $q \Leftrightarrow r$ premiss
3. Subderivation
   - 3.1 p assumption
   - 3.2 q 3.1,1,$\Leftrightarrow$-E-L
   - 3.3 r 3.2,2,$\Leftrightarrow$-E-L
4. $p \Rightarrow r$ 3,$\Rightarrow$-I

Sub-part (c) [1 mark]

For a sound and complete proof-theory, $A \models W$ if and only if $A \vdash W$