M.Sc. Mock Examinations 1996-97

Department of Computer Science

Part MSca

Mathematics for Information Processing

Time allowed: **30 minutes**

Silent non-programmable calculators may be used.
A set of 'Examination Notes' is attached. (*Actually they aren't. But they will be for the real exam.*)
Candidates should answer **all** questions from this paper.

1 (5 marks)

In this part of the question,

$$\begin{aligned}
S &\triangleq \{a, b, c\} \\
R_1 &: S \leftrightarrow S \\
R_1 &\triangleq \{\langle a, a\rangle, \langle b, b\rangle, \langle c, c\rangle\} \\
R_2 &: S \leftrightarrow S \\
R_2 &\triangleq \{\langle a, b\rangle, \langle b, a\rangle\} \\
R_3 &: S \leftrightarrow S \\
R_3 &\triangleq \{\langle a, b\rangle, \langle b, c\rangle, \langle a, c\rangle\} \\
R_4 &: S \leftrightarrow S \\
R_4 &\triangleq R_1 \cup R_2 \cup R_3
\end{aligned}$$

(a) [1 mark] Which, if any, of the four relations are functional?

(b) [1 mark] Which, if any, of the four relations are injective?

(c) [1 mark] Which, if any, of the four relations are one-to-many?

(d) [2 marks] Is R_4 an *equivalence relation*? If you think it is, give the partition that it forms on the set S. If you think it is not, explain why it is not.

2 (7 marks)

Suppose we have a set of people (e.g. Amy, Bill, Carl, etc.), called *People*, and a relation on *People*, called *Parent*, i.e.

$$Parent : People \leftrightarrow People$$

such that $\langle c, p\rangle \in Parent$ means that p is c's parent. [**Hint:** Be very clear about who is mapped to whom by the *Parent* relation.]

(a) [2 marks] Give a meaningful and simple paraphrase into English for each of the relations defined by the following expressions:

- $Parent^{-1}$
- $Parent \circ Parent$

(b) [3 marks] Write expressions that, when evaluated, would compute

- Amy's grandparents;
- whether Amy's parents are the same as Bill's.

(c) [2 marks] Complete the following definition of the unary, prefix operator *Ancestors*

$$Ancestors(p) \triangleq \ldots$$

which, given a person p, should return a set containing all of that person's ancestors.

3 (3 marks)

Recall that $\mathbb{Z}$ is the set of integers (negative, zero and positive) and $\mathbb{N}$ is the set of natural numbers (zero and positive integers).

Let the functions f_1 and f_2 be defined as follows:

$$\begin{array}{rcl} f_1 & : & \mathbb{Z} \rightarrow \mathbb{Z} \\ f_1(x) & \triangleq & x - 1 \\ f_2 & : & \mathbb{Z} \rightarrow \mathbb{N} \\ f_2(x) & \triangleq & x \uparrow 4 \end{array}$$

(a) [$1\frac{1}{2}$ marks]

- State whether f_1 is *surjective* or not. **Briefly justify** your answer.
- State whether f_2 is *surjective* or not. **Briefly justify** your answer.

(b) [$1\frac{1}{2}$ marks] The function f_3 is equal to the *composition* $f_2 \circ f_1$. Give a definition of f_3. PROBLEM: can give same answer either way: change 1 to 3, e.g.

Model Answers

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Question 1 (5 marks)

Sub-part (a) [1 mark]

R_1, R_2

Sub-part (b) [1 mark]

R_1, R_2

Sub-part (c) [1 mark]

None

Sub-part (d) [2 marks]

It is not. It must be reflexive, symmetric and transitive.

It is reflexive: every element is related to itself by virtue of the graph of R_1.

It is transitive: from the graph of R_3 the only relevant maplets are $a\ R_4\ b, b\ R_4\ c$ and $a\ R_4\ c$.

But it is not symmetric. Although R_2 is symmetric, unioning in $\langle a, c\rangle$ from graph of R_3 gives a maplet whose 'reverse' is not present.

(This last part is all that is needed for a correct answer, since all one must show is that one of the properties doesn't hold.)

Question 2 (7 marks)
Sub-part (a) [2 marks]

- $Parent^{-1}$: $c\ Parent\ p$ means c is a child of p (see the hint, and re-read the question if you thought otherwise), so the inverse is the 'parent of' relation.
- $Parent \circ Parent$: 'grandchild of'

Sub-part (b) [3 marks]

- $Im(Parent \circ Parent, \{Amy\})$
- $Im(Parent\ \{Amy\}) = Im(Parent, \{Bill\})$

Sub-part (c) [2 marks]

$Ancestors(p) \triangleq Im(TC(Parent), \{p\})$

Question 3 (3 marks)

Sub-part (a) [$1\frac{1}{2}$ **marks**]

- f_1 is surjective: every integer is the predecessor of some other integer;
- f_2 is not surjective: a counter-example is that 2 is not mapped to, i.e. there is no *integer* that when raised to the power 4 gives 2. [There are, of course, many other counter-examples.]

Sub-part (b) [$1\frac{1}{2}$ **marks**]

$$f_3 : \mathbb{Z} \rightarrow \mathbb{N}$$

$$f_3(x) \triangleq (x - 1) \uparrow 4$$