M.Sc. Mock Examinations 1996-97

Department of Computer Science

Part MSca

Mathematics for Information Processing

Time allowed: **30 minutes**

Silent non-programmable calculators may be used.
A set of 'Examination Notes' is attached. (*Actually they aren't. But they will be for the real exam.*)
Candidates should answer **all** questions from this paper.

1 (5 marks)

The following set names are used in parts of this question:

$\mathbb{N}$: the set of natural numbers (positive integers and zero)
$\mathbb{P}$: the set of positive integers (excludes zero)
$\mathbb{Z}$: the set of all integers (negative, zero and positive)

Evaluate the following expressions:

(a) [1 mark]
$(\{5, 6, 7\} \cap \{7, 5, 6, 8\}) \setminus$
$(\{x \mid x \in \mathbb{N} \text{ and } x < 6\} \cup \{x \mid x \in \mathbb{N} \text{ and } x \geq 8\})$

(b) [1 mark] $(\{6\} \cap \{2, 4\}) \in \emptyset$

(c) [1 mark] $\mathit{card}(\wp\{1, 2, 2\})$

(d) [1 mark] $\wp(\wp\{2\})$

(e) [1 mark] $(\wp(\mathbb{P} \cap (\mathbb{Z} \setminus \mathbb{N}))) \setminus \{\emptyset\}$

2 (5 marks)

In this question,

$$
\begin{aligned}
U &\triangleq \{b \mid b \text{ is a book}\} \\
A &\triangleq \{b \mid b \text{ is a paperback book}\} \\
B &\triangleq \{b \mid b \text{ is a book that I have read}\}
\end{aligned}
$$

Write expressions that, when evaluated, would compute

(a) [1 mark] the set of paperbacks that I have read;

(b) [1 mark] the set of hardbacks that I have read;

(c) [1 mark] the set of hardbacks that I haven't read;

(d) [1 mark] whether I have read more than 20 books; and

(e) [1 mark] whether all the books I have read are paperbacks.

3 (5 marks)

In this question, use the laws given in the 'Examination Notes' and the following additional law: for any sets SE_1 and SE_2,

$$SE_1 \setminus SE_2 = SE_1 \cap SE_2'$$

Prove the following algebraically: for any sets A_1 and A_2,

$$(A_1 \cap A_2)' \setminus (A_1 \cup A_2) = A_1' \cap A_2'$$

You **must** annotate each step of your proof with the algebraic law that is used in that step.

Model Answers

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Model Answers

Question 1 (5 marks)

Sub-part (a) [1 mark]

$\{6, 7\}$

Sub-part (b) [1 mark]

false

Sub-part (c) [1 mark]

4

Sub-part (d) [1 mark]

$\{\emptyset, \{\emptyset\}, \{\{2\}\}, \{\emptyset, \{2\}\}\}$

Sub-part (e) [1 mark]

$= \wp(\emptyset) \setminus \{\emptyset\} = \{\emptyset\} \setminus \{\emptyset\} = \emptyset$

Question 2 (5 marks)

Sub-part (a) [1 mark]

$A \cap B$

Sub-part (b) [1 mark]

$A' \cap B$ or $B \setminus A$

Sub-part (c) [1 mark]

$A' \cap B'$ or $A' \setminus B$

Sub-part (d) [1 mark]

$card(B) > 20$

Sub-part (e) [1 mark]

$B \subseteq A$ or $A \cap B = B$

Question 3 (5 marks)

$$
\begin{array}{lll}
\text{LHS} & = (A_1 \cap A_2)' \setminus (A_1 \cup A_2) & \\
& = (A_1 \cap A_2)' \cap (A_1 \cup A_2)' & \text{(defn of set diff)} \\
& = (A_1' \cup A_2') \cap (A_1 \cup A_2)' & \text{(De Morgan)} \\
& = (A_1' \cup A_2') \cap (A_1' \cap A_2') & \text{(De Morgan)} \\
& = (A_1' \cap A_2') \cap (A_1' \cup A_2') & \text{(comm)} \\
& = ((A_1' \cap A_2') \cap A_1') \cup ((A_1' \cap A_2') \cap A_2') & \text{(distrib)} \\
& = (A_1' \cap (A_1' \cap A_2')) \cup ((A_1' \cap A_2') \cap A_2') & \text{(comm)} \\
& = ((A_1' \cap A_1') \cap A_2') \cup ((A_1' \cap A_2') \cap A_2') & \text{(assoc)} \\
& = (A_1' \cap A_2') \cup ((A_1' \cap A_2') \cap A_2') & \text{(idemp)} \\
& = (A_1' \cap A_2') \cup (A_1' \cap (A_2' \cap A_2')) & \text{(assoc)} \\
& = (A_1' \cap A_2') \cup (A_1' \cap A_2') & \text{(idemp)} \\
& = A_1' \cap A_2' & \text{(idemp)} \\
& = \text{RHS} &
\end{array}
$$