061311R

B.Sc. and B.A. Examinations 1997

Department of Computer Science

Part I Resit

Mathematics for Information Processing

Time allowed: **One-and-a-half hours**

Silent non-programmable calculators may be used.
A set of 'Examination Notes' is attached.
Candidates should answer **all** questions from this paper.

In this paper the following set names are used throughout:

$$
\begin{aligned}
Suits &\triangleq \{Clubs, Diamonds, Hearts, Spades\} \\
RedSuits &\triangleq \{Diamonds, Hearts\} \\
BlackSuits &\triangleq \{Clubs, Spades\} \\
Values &\triangleq \{A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K\} \\
Players &\triangleq \{Ann, Ben, Col\} \\
Cards &\triangleq Values \times Suits
\end{aligned}
$$

1 (10 marks)

(i) [3 marks] Give *extensional* definitions of each of the following sets:

(a) [$\frac{1}{2}$ marks] $RedSuits \cup BlackSuits$

(b) [$\frac{1}{2}$ marks] $RedSuits \cap BlackSuits$

(c) [$\frac{1}{2}$ marks] $Suits \setminus BlackSuits$

(d) [$\frac{1}{2}$ marks] $BlackSuits \setminus Suits$

(e) [$\frac{1}{2}$ marks] $Suits \cup RedSuits$

(f) [$\frac{1}{2}$ marks] $RedSuits \cap Suits$

(ii) [2 marks] State whether the following are true or false:

(a) [$\frac{1}{2}$ marks] $RedSuits \in \wp(Suits)$

(b) [$\frac{1}{2}$ marks] $RedSuits \subseteq \wp(Suits)$

(c) [$\frac{1}{2}$ marks] $\emptyset \in \wp(Suits)$

(d) [$\frac{1}{2}$ marks] $\emptyset \subseteq \wp(Suits)$

(iii) [5 marks] Prove the following algebraically using the laws given in the 'Examination Notes'. You **must** annotate each step of your proof with the algebraic law that is used in that step.

For any sets A and B,

$$(A' \cup B') \cap B = A' \cap B$$

2 (20 marks)

(i) [3 marks] Cut is a relation from $Players$ to $Values$. It maps a player to the value of the card that s/he turns up in a cut of a deck of cards:

$$\begin{array}{rcl} Cut & : & Players \leftrightarrow Values \\ Cut & \triangleq & \{\langle Ann, 6\rangle, \langle Ben, K\rangle\} \end{array}$$

- Is Cut *total* or *partial*?
- Is it *surjective* or not?
- Is it *functional* or not?
- Is it *injective* or not?
- Is it *one-to-one, one-to-many, many-to-one* or *many-to-many*?
- Is it a *bijection*?

(ii) [11 marks] $Deal$ is a relation from $Players$ to $Cards$. It maps a player to cards that s/he has been dealt:

$$Deal : Players \leftrightarrow Cards$$

e.g. if Ann has been dealt the 2 of Hearts and the King of Spades, then $\langle Ann, \langle 2, Hearts\rangle\rangle$ and $\langle Ann, \langle K, Spades\rangle\rangle$ are tuples in the graph of $Deal$.

Write expressions involving the $Deal$ relation that, when evaluated, would compute:

(a) [2 marks] the *set* of all cards dealt to Ben;

(b) [2 marks] the player who has been dealt the Queen of Spades;

(c) [2 marks] whether all cards have been dealt;

(d) [2 marks] whether Ben has been dealt more cards that Col has; and

(e) [3 marks] the *set* of all cards dealt to players who turned up a 6 or a 7 in a cut of the deck.

(iii) [6 marks] Consider the following relation on $Suits$

$$WMT : Suits \leftrightarrow Suits$$
$$WMT \triangleq \{\langle Spades, Hearts\rangle, \langle Hearts, Diamonds\rangle, \langle Diamonds, Clubs\rangle\}$$

(WMT can be thought of as the 'worth more than' relation.)

(a) [3 marks] Carefully **explain** why WMT is *not* an *ordering*.

(b) [3 marks] Relation WMT' is just like WMT except that its graph has fewer tuples. Specifically:

$$WMT' = WMT \setminus \{\langle Hearts, Diamonds\rangle\}$$

Is WMT' an *ordering*? **Justify** your answer.

3 (20 marks)

(i) [3 marks] In this part of the question,

$$p \triangleq \text{Ann is dealt the King of Hearts}$$
$$q \triangleq \text{Ben is dealt the Queen of Hearts}$$

Express the following statements as *well-formed formulae of propositional logic.*

(a) [1 mark] Ann is dealt the King of Hearts if Ben is dealt the Queen of Hearts.