061311

THE UNIVERSITY *of York*

M.Sc., B.Sc. and B.A. Examinations 1996-97

DEPARTMENT OF COMPUTER SCIENCE

Part Ia, MScIPa

Mathematics for Information Processing

Time allowed: **One-and-a-half hours**

Silent non-programmable calculators may be used.
A set of 'Examination Notes' is attached.
Candidates should answer **all** questions from this paper.

1 (10 marks)

(i) [4 marks] Let $S_1 \triangleq \{b, a, a\}$ and $S_2 \triangleq \{a, b, \{b\}, \{a\}, \{a, b\}\}$.

(a) [2 marks] Is $S_1 \in S_2$? Explain your answer.

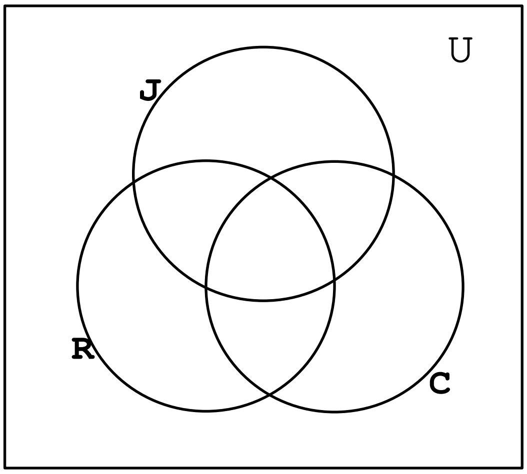
(b) [2 marks] Is $S_1 \subseteq S_2$? Explain your answer.

(ii) [6 marks] In a class of 66 people, it was found that 10 people liked Jazz, 42 people liked Rock music and 29 people liked Classical music; there was no-one who disliked music. Of the 66, 7 liked both Jazz and Rock, 5 liked both Rock and Classical music, and 5 liked both Jazz and Classical music.

Let J, R and C denote the sets of Jazz, Rock and Classical music fans respectively. Redraw the following Venn diagram, inserting the correct numbers to show the cardinality of each of the eight regions of the diagram. Include some sentences in your answer that explain your working.

You will need to use the following law to obtain your answer: for any sets A_1, A_2 and A_3,

$$\begin{aligned}
&\mathit{card}((A_1 \cup A_2) \cup A_3) = \\
&\quad \mathit{card}(A_1) + \mathit{card}(A_2) + \mathit{card}(A_3) \\
&\quad -\mathit{card}(A_1 \cap A_2) - \mathit{card}(A_1 \cap A_3) - \mathit{card}(A_2 \cap A_3) \\
&\quad +\mathit{card}((A_1 \cap A_2) \cap A_3)
\end{aligned}$$



2 (20 marks)

(i) [9 marks] In this part of the question,

$$\mathit{Music} \triangleq \{\mathit{Jazz, Rock, Classical, Folk}\}$$

Ann's music tastes are captured by the following relation:

$$\begin{aligned} LMT_A &: \mathit{Music} \leftrightarrow \mathit{Music} \\ LMT_A &\triangleq \{\langle \mathit{Jazz, Rock}\rangle, \langle \mathit{Rock, Classical}\rangle, \langle \mathit{Classical, Folk}\rangle\} \end{aligned}$$

(where $\langle x, y\rangle \in LMT_A$ can be thought of as stating that Ann Likes music type x More Than music type y).

(a) [3 marks]

- Is LMT_A a *total* or *partial* relation?
- Is it *surjective* or not?
- Is it *functional* or not?
- Is it *injective* or not?
- Is it *one-to-one*, *one-to-many*, *many-to-one* or *many-to-many*?
- Is it a *bijection*?

(b) [3 marks] Carefully explain why LMT_A is *not* an *ordering*.

(c) [3 marks] Ben's music tastes are given by the relation LMT_B which is just like LMT_A except that its graph has fewer tuples. Specifically,

$$LMT_B = LMT_A \setminus \{\langle \mathit{Rock, Classical}\rangle\}$$

Is LMT_B an *ordering*? **Justify** your answer.

(ii) [11 marks] Col has defined a simple 'database' to keep track of his CD collection. He assigns each of his CD's a unique *identifying number* (drawn from $\mathbb{P}$, the positive integers) and then associates the CD title and the artiste with these numbers. The sets and relations he uses are shown below. (Only a few example members of the extensions of the sets and relations are shown.)

$$
\begin{aligned}
\mathit{Titles} &\triangleq \{\text{“}\mathit{Different\ Class}\text{”}, \text{“}\mathit{Parklife}\text{”}, \ldots\} \\
\mathit{Artistes} &\triangleq \{\mathit{Blur}, \mathit{Oasis}, \mathit{Pulp}, \ldots\} \\
\mathit{CDTitle} &: \mathbb{P} \to \mathit{Titles} \\
\mathit{CDTitle} &\triangleq \{\langle 1, \text{“}\mathit{Different\ Class}\text{”}\rangle, \langle 2, \text{“}\mathit{Parklife}\text{”}\rangle, \ldots\} \\
\mathit{CDArtiste} &: \mathbb{P} \to \mathit{Artistes} \\
\mathit{CDArtiste} &\triangleq \{\langle 1, \mathit{Pulp}\rangle, \langle 2, \mathit{Blur}\rangle, \ldots\}
\end{aligned}
$$

(The definitions show, for example, that CD number 1 is the excellent “Different Class” by Pulp, CD number 2 is the slightly less excellent “Parklife” by Blur, and so on.)

Write expressions that, when evaluated, would compute

(a) [2 marks] the identifying numbers of all CDs that are by either Blur or Oasis;

(b) [3 marks] whether Col owns more Blur CDs than Oasis CDs;

(c) [3 marks] the identifying numbers of all CDs whose artistes are in the database but whose titles are not, or whose titles are in the database but whose artistes are not;

(d) [3 marks] the titles of all CDs in Col’s collection that are by Oasis.

3 (20 marks)

(i) [2 marks] In this part of the question,

$$p \triangleq \text{Johnny likes Jazz}$$
$$q \triangleq \text{Katie likes Johnny}$$

Express the following statements as *well-formed formulae of propositional logic.*

(a) [1 mark] Katie likes Johnny if Johnny doesn't like Jazz.

(b) [1 mark] Katie likes Johnny unless Johnny likes Jazz.

(ii) [13 marks]

(a) [4 marks] Use a *truth table* to show that

$$\{p, (p \vee q) \Leftrightarrow r\} \models r$$

(b) [4 marks] Use the rules of the *natural deduction* system given in the 'Examination Notes' to show that

$$\{p, (p \vee q) \Leftrightarrow r\} \vdash r$$

You **must** annotate each step of your derivation with the inference rule that the step uses.

(c) [5 marks] Prove the following *algebraically* using the *laws* given in the 'Examination Notes'. You **must** annotate each step of your proof with the algebraic law that is used in that step.

$$(\neg p \vee \neg q) \wedge q \equiv \neg p \wedge q$$

(iii) [5 marks] In this part of the question, use the following 'key':

$s(x)$: x is a student
$i(x)$: x is a musical instrument
$p(x, y)$: x can play y

(s and i are unary predicate symbols, and p is a binary predicate symbol.)

(a) [2 marks] Give a natural English paraphrase for the following formula of first-order predicate logic:

$$\exists x \exists y((i(x) \land s(y) \land p(y, x)) \Leftrightarrow \forall z(s(z) \Rightarrow p(z, x)))$$

(b) [3 marks] Translate the following English statements into well-formed formulae of first-order predicate logic:

- Every student can play at least one instrument.
- At least one student can play all instruments.

EXAMINATION NOTES

Set Theory

For any sets SE, SE_1, SE_2 and SE_3,...

Commutativity of $\cup$ and $\cap$

$$SE_1 \cup SE_2 = SE_2 \cup SE_1$$
$$SE_1 \cap SE_2 = SE_2 \cap SE_1$$

Associativity of $\cup$ and $\cap$

$$(SE_1 \cup SE_2) \cup SE_3 = SE_1 \cup (SE_2 \cup SE_3)$$
$$(SE_1 \cap SE_2) \cap SE_3 = SE_1 \cap (SE_2 \cap SE_3)$$

Distributivity of $\cup$ over $\cap$, and $\cap$ over $\cup$

$$SE_1 \cup (SE_2 \cap SE_3) = (SE_1 \cup SE_2) \cap (SE_1 \cup SE_3)$$

$$SE_1 \cap (SE_2 \cup SE_3) = (SE_1 \cap SE_2) \cup (SE_1 \cap SE_3)$$

De Morgan's laws

$$(SE_1 \cup SE_2)' = SE_1' \cap SE_2'$$
$$(SE_1 \cap SE_2)' = SE_1' \cup SE_2'$$

Turn over.

Idempotence of $\cup$ and $\cap$

$$SE \cup SE = SE$$

$$SE \cap SE = SE$$

Identities

$$\emptyset \cup SE = SE$$

$$U \cup SE = U$$

$$\emptyset \cap SE = \emptyset$$

$$U \cap SE = SE$$

Involution

$$(SE')' = SE$$

Complement laws

$$SE \cup SE' = U$$

$$SE \cap SE' = \emptyset$$

$$U' = \emptyset$$

$$\emptyset' = U$$

Logic

For any propositions W, W_1, W_2 and W_3,...

Commutativity of $\wedge$ and $\vee$

$$W_1 \wedge W_2 \equiv W_2 \wedge W_1$$
$$W_1 \vee W_2 \equiv W_2 \vee W_1$$

Associativity of $\wedge$ and $\vee$

$$(W_1 \wedge W_2) \wedge W_3 \equiv W_1 \wedge (W_2 \wedge W_3)$$
$$(W_1 \vee W_2) \vee W_3 \equiv W_1 \vee (W_2 \vee W_3)$$

Distributivity of $\wedge$ over $\vee$, and of $\vee$ over $\wedge$

$$W_1 \wedge (W_2 \vee W_3) \equiv (W_1 \wedge W_2) \vee (W_1 \wedge W_3)$$
$$W_1 \vee (W_2 \wedge W_3) \equiv (W_1 \vee W_2) \wedge (W_1 \vee W_3)$$

De Morgan's laws

$$\neg(W_1 \wedge W_2) \equiv \neg W_1 \vee \neg W_2$$
$$\neg(W_1 \vee W_2) \equiv \neg W_1 \wedge \neg W_2$$

Idempotence of $\wedge$ and $\vee$

$$W \wedge W \equiv W$$
$$W \vee W \equiv W$$

Turn over.

Identities

$$t \wedge W \equiv W$$

$$f \wedge W \equiv f$$

$$t \vee W \equiv t$$

$$f \vee W \equiv W$$

Involution

$$\neg\neg W \equiv W$$

Complement laws

$$W \wedge \neg W \equiv f$$

$$W \vee \neg W \equiv t$$

$$\neg t \equiv f$$

$$\neg f \equiv t$$

Definition of biconditional

$$W_1 \Leftrightarrow W_2 \equiv (W_1 \Rightarrow W_2) \wedge (W_2 \Rightarrow W_1)$$

Definition of conditional

$$W_1 \Rightarrow W_2 \equiv \neg W_1 \vee W_2$$

Contrapositive law

$$W_1 \Rightarrow W_2 \equiv \neg W_2 \Rightarrow \neg W_1$$

Definition of existential

$$\exists X W \equiv \neg \forall X \neg W$$

Definition of universal

$$\forall X W \equiv \neg \exists X \neg W$$

Some inference rules of a natural deduction system

$\wedge$-INTRODUCTION
$$\frac{W_1, W_2}{W_1 \wedge W_2}$$

$\wedge$-ELIMINATION-LEFT
$$\frac{W_1 \wedge W_2}{W_1}$$

$\wedge$-ELIMINATION-RIGHT
$$\frac{W_1 \wedge W_2}{W_2}$$

$\Rightarrow$-ELIMINATION
$$\frac{W_1, W_1 \Rightarrow W_2}{W_2}$$

$\vee$-INTRODUCTION-RIGHT
$$\frac{W_1}{W_1 \vee W_2}$$

$\vee$-INTRODUCTION-LEFT
$$\frac{W_2}{W_1 \vee W_2}$$

Turn over.

⇔-ELIMINATION-LEFT

$$\frac{W_1 \Leftrightarrow W_2, W_1}{W_2}$$

⇔-ELIMINATION-RIGHT

$$\frac{W_1 \Leftrightarrow W_2, W_2}{W_1}$$

¬-ELIMINATION

$$\frac{\neg\neg W}{W}$$

⇒-INTRODUCTION

$$\frac{\frac{W_1}{W_2}}{W_1 \Rightarrow W_2}$$

REPETITION

$$\frac{W}{W}$$