061311

THE UNIVERSITY *of York*

M.Sc., B.Sc. and B.A. Examinations 1996-97

DEPARTMENT OF COMPUTER SCIENCE

Part Ia, MScIPa

Mathematics for Information Processing

Time allowed: **One-and-a-half hours**

Silent non-programmable calculators may be used.
A set of 'Examination Notes' is attached.
Candidates should answer **all** questions from this paper.

In this paper the following set names are used throughout:

[At this point, the names of any sets used in the exam paper will be given. This will only appear if there are sets that are used in more than one question.]

1 (10 marks)

[A question on set theory.]

2 (20 marks)

[A question on relations and functions (but also requiring a knowledge of set theory).]

3 (20 marks)

[A question on propositional logic and predicate logic (but also requiring a knowledge of set theory, relations and functions.)]

EXAMINATION NOTES

Set Theory

For any sets SE, SE_1, SE_2 and SE_3,...

Commutativity of $\cup$ and $\cap$

$$SE_1 \cup SE_2 = SE_2 \cup SE_1$$

$$SE_1 \cap SE_2 = SE_2 \cap SE_1$$

Associativity of $\cup$ and $\cap$

$$(SE_1 \cup SE_2) \cup SE_3 = SE_1 \cup (SE_2 \cup SE_3)$$

$$(SE_1 \cap SE_2) \cap SE_3 = SE_1 \cap (SE_2 \cap SE_3)$$

Distributivity of $\cup$ over $\cap$, and $\cap$ over $\cup$

$$SE_1 \cup (SE_2 \cap SE_3) = (SE_1 \cup SE_2) \cap (SE_1 \cup SE_3)$$

$$SE_1 \cap (SE_2 \cup SE_3) = (SE_1 \cap SE_2) \cup (SE_1 \cap SE_3)$$

De Morgan's laws

$$(SE_1 \cup SE_2)' = SE_1' \cap SE_2'$$

$$(SE_1 \cap SE_2)' = SE_1' \cup SE_2'$$

Turn over.

Idempotence of $\cup$ and $\cap$

$$SE \cup SE = SE$$

$$SE \cap SE = SE$$

Identities

$$\emptyset \cup SE = SE$$

$$U \cup SE = U$$

$$\emptyset \cap SE = \emptyset$$

$$U \cap SE = SE$$

Involution

$$(SE')' = SE$$

Complement laws

$$SE \cup SE' = U$$

$$SE \cap SE' = \emptyset$$

$$U' = \emptyset$$

$$\emptyset' = U$$

Logic

For any propositions W, W_1, W_2 and W_3,...

Commutativity of $\wedge$ and $\vee$

$$W_1 \wedge W_2 \equiv W_2 \wedge W_1$$
$$W_1 \vee W_2 \equiv W_2 \vee W_1$$

Associativity of $\wedge$ and $\vee$

$$(W_1 \wedge W_2) \wedge W_3 \equiv W_1 \wedge (W_2 \wedge W_3)$$
$$(W_1 \vee W_2) \vee W_3 \equiv W_1 \vee (W_2 \vee W_3)$$

Distributivity of $\wedge$ over $\vee$, and of $\vee$ over $\wedge$

$$W_1 \wedge (W_2 \vee W_3) \equiv (W_1 \wedge W_2) \vee (W_1 \wedge W_3)$$
$$W_1 \vee (W_2 \wedge W_3) \equiv (W_1 \vee W_2) \wedge (W_1 \vee W_3)$$

De Morgan's laws

$$\neg(W_1 \wedge W_2) \equiv \neg W_1 \vee \neg W_2$$
$$\neg(W_1 \vee W_2) \equiv \neg W_1 \wedge \neg W_2$$

Idempotence of $\wedge$ and $\vee$

$$W \wedge W \equiv W$$
$$W \vee W \equiv W$$

Turn over.

Identities

$$t \wedge W \equiv W$$

$$f \wedge W \equiv f$$

$$t \vee W \equiv t$$

$$f \vee W \equiv W$$

Involution

$$\neg\neg W \equiv W$$

Complement laws

$$W \wedge \neg W \equiv f$$

$$W \vee \neg W \equiv t$$

$$\neg t \equiv f$$

$$\neg f \equiv t$$

Definition of biconditional

$$W_1 \Leftrightarrow W_2 \equiv (W_1 \Rightarrow W_2) \wedge (W_2 \Rightarrow W_1)$$

Definition of conditional

$$W_1 \Rightarrow W_2 \equiv \neg W_1 \vee W_2$$

Contraposive law

$$W_1 \Rightarrow W_2 \equiv \neg W_2 \Rightarrow \neg W_1$$

Definition of existential

$$\exists X W \equiv \neg \forall X \neg W$$

Definition of universal

$$\forall X W \equiv \neg \exists X \neg W$$

Some inference rules of a natural deduction system

$\wedge$-INTRODUCTION
$$\frac{W_1, W_2}{W_1 \wedge W_2}$$

$\wedge$-ELIMINATION-LEFT
$$\frac{W_1 \wedge W_2}{W_1}$$

$\wedge$-ELIMINATION-RIGHT
$$\frac{W_1 \wedge W_2}{W_2}$$

$\Rightarrow$-ELIMINATION
$$\frac{W_1, W_1 \Rightarrow W_2}{W_2}$$

$\vee$-INTRODUCTION-RIGHT
$$\frac{W_1}{W_1 \vee W_2}$$

$\vee$-INTRODUCTION-LEFT
$$\frac{W_2}{W_1 \vee W_2}$$

Turn over.

⇔-ELIMINATION-LEFT

$$\frac{W_1 \Leftrightarrow W_2, W_1}{W_2}$$

⇔-ELIMINATION-RIGHT

$$\frac{W_1 \Leftrightarrow W_2, W_2}{W_1}$$

¬-ELIMINATION

$$\frac{\neg\neg W}{W}$$

⇒-INTRODUCTION

$$\frac{\frac{W_1}{W_2}}{W_1 \Rightarrow W_2}$$

REPETITION

$$\frac{W}{W}$$