

On Motifs and Functional Blocks in Technological Networks

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One primary focus of complex systems research has been the topological analysis of the network underlying a complex system. One key question is the size and structure/function of the underlying building blocks. Two proposed building blocks are called motif and functional module. A *motif* has been defined as a subgraph that occurs significantly more frequently in real-world networks than expected by chance alone [5]. The observed over-representation of motifs has been interpreted as a manifestation of functional constraints and design principles that have shaped network architectures at the local level. In contrast, a *functional module* or block in a network is a set of nodes that have strong interactions and a distinct function [1]. We distinguish the two by emphasizing small size and recurrence for motifs, endowing modules with larger size, and perhaps a composition dominated by interconnected motifs [7]. However, the relation between motifs and functional blocks lacks adequate precision, and has not received sufficient experimental analysis.

This article addresses the relation between motifs and functional modules (or blocks) for a class of complex network, electronic circuits, where nodes are electronic components (e.g. logic gates and flip-flops in digital circuits), and edges are wires [2]. We compared a library of functional modules used by engineers for circuit design with the modules that are automatically identified using motif-finding algorithms [6]. This library of circuit functional blocks was either reverse-engineered [3] or identified from IC databooks, cell libraries or textbooks. Given this library, we calculated the relative frequency of the occurrence of the functional modules; we also compared the topologies and instances of motifs and functional blocks.

Comparing the hand-generated functional modules with the experimentally-computed motifs showed that: (1) functional modules are much larger than the 3-4 node motifs proposed in [4], and (2) a functional module typically consists of a collection of several 3-4 node motifs. We argue that, for electronic circuits, the statistical over-representation of 3-4 node motifs is due to functional modules consisting of motif-clusters, and not that motifs, in and of themselves, play a significant functional role in a circuit. More generally, the common functional modules discovered in benchmark circuits can be considered as “large motifs” themselves.

We further examined the frequency of occurrence of structures corresponding to circuit building blocks in the random graph ensemble produced in motif detection. We found that the common functional modules discovered in benchmark circuits are unlikely to be generated by the generalized random graph model [5].

Finally, we analyzed the significance profile (SP) [4] of benchmark circuits. The SP measure aids the comparison of networks of different sizes, because motifs in large networks tend to display higher Z-scores than motifs in small networks. We found that circuits having similar functions are highly correlated in terms of the SP. Hence, SP analysis is an effective approach for comparing the local structure and function of benchmark circuits.

References

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